How to close the skill gap?

Parental Background and Children's Skill Development in Indonesia*

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Abstract

Preexisting inequalities in socioeconomic status can drive differences in children's cognitive skill development and parents' reactions to child development policies influencing policy effectiveness. To analyze the role of parental background and investments (nutrition diversity and schooling expenditure) in this process, I estimate a dynamic structural model using data from Indonesia. Using the model, I simulate three policies: unconditional cash transfers, nutrition, and schooling price subsidies. To compare their long-run effects on adult skills, I account for parents adjusting their investment behavior in response to policies. Given the same cost, a) subsidizing food prices is more effective than subsidizing schooling expenditure, and b) both are more effective than cash transfers. As I find nutrition and schooling to be complements, a price decrease incentivizes parents to increase both inputs. With cash transfers, parents also increase investments but increase consumption relatively more as price incentives do not change. Nutrition subsidies reduce inequality most effectively, as parents with lower education react stronger to food price changes and, consequently, increase child investments more than parents with higher education. They do so as they spend a larger share of investments on nutrition. Further, nutrition subsidies implemented alone are more cost-effective than any combination of the three policies.

Keywords: Child development, Human capital, Inequality, Cognitive skills, Human capital production functions, Development, Cash transfers, Indonesia

JEL Codes: I24, I38, J24

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1 Introduction

Two-thirds of children globally do not obtain basic skills, and a vast majority of them reside in low- and middle-income countries (Gust, Hanushek and Woessmann, 2022).[1] Within these countries, low cognitive skills are concentrated among children from poorer backgrounds. Early in life, they display lower skill levels than children from wealthier households, which translates into a persistent adult skill gap. This gap results in lower intergenerational mobility and higher inequality (Attanasio, Meghir and Nix (2020)). Simultaneously, there exist significant disparities in parental investments by socioeconomic background. In Indonesia, parents with high school education spend on average more than triple in their child's schooling and invest 15% more in nutrition diversity than parents with no education - who earn less than twice of their income.[2] How much of the adult skill gap is driven by these investment differences compared to parental characteristics? Why does investment behavior vary by socioeconomic status? Are some parents more productive in investing or less resource constrained? Answering these questions is crucial to design effective policies to reduce the gap in adult skills and increase overall skill levels. Different investment behavior by socioeconomic status might lead to parents reacting differently to policies. If so, policies will vary in the degree to which they reduce inequality in skills. Knowing why and when parents invest differently allows to take their response to policies into account and assess the long-run effects of policies on skill levels and inequality.

Therefore, in this paper, I explicitly model parental investment choices and examine how cognitive skill differences transmit from childhood to adulthood outcomes in the setting of Indonesia. Using a dynamic structural model, I quantify the role of parental background and investments (nutrition diversity and schooling expenditure) in skill development. I extend existing frameworks for child development, as Del Boca, Flinn and Wiswall (2014) and Caucutt et al. (2020), by quantifying the impact of parental decisions on nutrition diversity in children's cognitive development. In doing so, I adapt the framework to a low- and middle-income country setting. Here, resources are scarce, and food insecurity plays a prominent role in child development (Aurino, Fledderjohann and Vellakkal (2019), Galasso, Weber and Fernald (2019)). While Attanasio et al. (2020) and Attanasio, Meghir and Nix (2020) estimate children's skill formation in a low- and middle-income country setting, they do not explicitly model parental choices following Cunha, Heckman and Schennach (2010).

[1] Basic skills are equivalent to PISA Level 1 skills (able to identify information and carry out routine procedures according to direct instructions in explicit situations).

[2] Author's calculations with data from the Indonesian Family Life Survey (IFLS), supplied by the RAND cooperation. For details, see Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009) and Strauss, Witoelar and Sikoki (2016). Nutrition diversity is measured as the number of food groups consumed.

By modeling parental choices, I can evaluate policies' long-run effects, carefully controlling for parental responses. I focus on evaluating cash transfers, food and schooling price subsidies, and their joint implementation. For a careful evaluation of these policies, it is crucial that I estimate the substitutability of schooling and nutrition inputs. The degree of substitutability determines how parents increase investment inputs given price subsidies or budget increases and how much cognitive skills increase in the long run.

I employ and estimate a dynamic structural model where parents face a trade-off between consumption, saving, and investing in their child's skills and are constrained by their income and assets.[3] Parents' socioeconomic background shapes their choices via three key mechanisms, and I incorporate them to differ in influence by childhood period. First, preferences for cognitive skills are allowed to vary by parental education. Parents with lower education might value cognitive skills more as they wish their children to have a better life than them. Second, parental choices are constrained by income and assets, which differ by parental education level. Third, I allow for differences in the technology of skill production. Parents with higher education might be more productive in converting the same level of investments into future skills because they can, for instance, encourage learning during playing. They also might be more productive with schooling expenditure by, for example, being able to support their children with homework. These productivity advantages would allow some parents to invest less and yield the same outcome as parents who invest more.

Using this framework, I estimate children's skill formation for each childhood period. I exploit a rich panel data set, the Indonesian Family Life Survey (IFLS). The IFLS follows a large sample of children over time, recording several measures for cognitive skills and parents' investment choices and characteristics. This feature allows me to account for the time-varying impact of parental characteristics and parenting skills and identify production technology and preferences. Further, I identify if parental investments, nutrition diversity, and schooling expenditure are substitutes or complements using available time and regional variation in food prices. If substitutes, parents increase the demand for inputs which drop in price and substitute the other. However, if inputs are complements, a price decrease in food increases both inputs. This mechanism influences how parents react to policies and their effectiveness. Hence, I can use the model in simulations to quantify the drivers of the adult skill gap and the long-run effects of policies.

I target the lowest 20% of the income distribution in my policy experiments as income

[3] Different to Del Boca, Flinn and Wiswall (2014) or Caucutt et al. (2020), I do not model the time parents spend with their children but focus on schooling and introduce nutrition diversity to the model. I focus my analysis on the later periods of childhood as Del Boca, Flinn and Wiswall (2014) find time to matter less than in early childhood. This might be extended for the evaluation of cash transfers as parental time allocation is highly sensitive to participation in transfer programs (Flores, 2021).

plays a significant role in the skill gap. My simulations show that subsidizing schooling or nutrition prices is more effective than unconditional cash transfers for the same costs.[4] Food price subsidies increase adult skills on average by 0.04 SD and a schooling subsidy by 0.03 SD, while cash transfers have negligible effects. While cash transfers help to lift income constraints, price subsidies change the proportion of investment inputs. As I find nutrition and schooling to be complements, lowering one input price leads to an increase in both inputs.[5] If I compare impacts across the income distribution, cash transfers and nutrition subsidies' impacts decrease with income, while schooling impacts slightly increase. This pattern indicates that parents with low income are significantly more budget constrained and less effective at using schooling investments productively compared to nutrition investments. They spend a higher share of their investment on nutrition resulting in them reacting stronger to nutrition subsidies. Hence, to reduce inequality, nutrition subsidies are the most cost-effective policy. They are also more cost-effective than combining different policies.

Related Literature I contribute to the literature in a three-fold way. First, I add to the research on nutrition and its importance for child development by modeling nutrition diversity as a separate investment input. Doing so, I compare policies accounting for parental responses and identify changes in nutrition and schooling investments due to food price changes. Interventions like food stamp allocation, nutrition supplementation, and cash transfers reduce stunting (extremely low height-by-age), and early childhood stunting has been shown to decrease cognitive skills (Sánchez (2017), Bailey et al. (2020), Galasso, Weber and Fernald (2019), Carneiro et al. (2021)). Nutrition diversity has long run-effects, as early childhood interventions increasing protein intake have been found to result in higher adult cognitive skills (Hoddinott et al. (2008), Behrman, Hoddinott and Maluccio (2020)). However, nutrition affects outcomes not only early in life. School meal programs show significant effects for poorer children on test scores in middle childhood (Aurino et al. (2020), Frisvold (2015)). Impacts increase if school meals are designed to be healthy, emphasizing the importance of diversity (Belot and James, 2011). Further evidence shows that children are negatively affected by higher food prices, especially protein price increases (see Vellakkal et al. (2015), Kandpal et al. (2016), Filmer et al. (2021) and Headey, Hirvonen and Hoddinott (2018)).[6] My results complement these findings as parents increase nutrition diversity with lower food

[4] Cash transfer size corresponds to 3% of the mean annual income of the lowest 20% of the income distribution.

[5] The percent increase of the targeted input is higher than of the other input. However, the other input increases as well, and therefore total investments.

[6] Kandpal et al. (2016) and Filmer et al. (2021) show that by a cash transfer in the Philippines stunting decreases via higher protein intake. In comparison, ineligible children are negatively affected in regions with higher protein prices (an association also found by Headey, Hirvonen and Hoddinott (2018) for protein prices and Vellakkal et al. (2015) for food prices in general).

prices leading to higher cognitive skills. However, I depart from the literature by analyzing the co-movement of nutrition and schooling investments. I find schooling expenditure also increases, magnifying food price subsidies' effects.

Second, I contribute to the literature on long-run policy evaluations in developing countries by comparing policies taking into account parental responses. Summarizing the existing evidence, Bouguen et al. (2019) conclude that direct investments in health, cognitive stimulation in early childhood, scholarships, and in some cases, conditional cash transfers have positive effects.[7] My contribution lies in simulating the different combinations and synergies of a collection of policies at different points in childhood. By this, I add to the literature on the use of structural models evaluating child development policies (Todd and Wolpin (2006), Duflo (2012), Daruich (2018), Bobba et al. (2021)). I extend this literature by looking, in particular, at reactions to policies subsidizing investment prices. Food price subsidies have been found to have mixed effects on nutrition diversity. Jensen and Miller (2018) do not find any increases for a staple subsidy in China. In contrast, Kaul (2018) and Krishnamurthy, Pathania and Tandon (2017) find increases in nutritional diversity, especially of young children, for a price subsidy in India. I extend the literature by modeling several dimensions of parental investment responses to price changes. Additionally, I can focus on the long-run effects on cognitive skills as I estimate skill formation up to adulthood. This feature allows me to model the 'missing middle years' of childhood, primary education, a period which is less researched (Almond, Currie and Duque, 2018). How skill changes by policies translate into middle childhood and how these indicators predict adult outcomes would help compare early life interventions with adolescent ones.

Third, I use data from a lower middle-income country to estimate skill production functions. Parents in low and middle-income countries operate under stronger income constraints, and food scarcity plays a bigger role than in high-income countries. Most of the existing literature on estimating skill production functions uses data from high-income countries (Todd and Wolpin (2007), Bernal (2008), Cunha and Heckman (2008), Cunha, Heckman and Schennach (2010), Del Boca, Flinn and Wiswall (2014), Lee and Seshadri (2019), Caucutt et al. (2020)). Exceptions are, Villa (2017) for the Philippines, Attanasio, Meghir and Nix (2020) for India and Attanasio et al. (2020) for Colombia. However, these studies pool investments and do not model inputs like nutrition separately. Thus, parental choices are not modeled explicitly, and their behavior adaptations to policies cannot be simulated. By modeling nutrition and

[7] The evidence for the effects of cash transfers on adult outcomes is mixed (see Molina Millán et al. (2019) for a summary). Particularly, for unconditional cash transfers, the long-term evidence is scarce due to fewer trials available (exceptions are Araújo, Bosch and Schady (2018) and Baird, McIntosh and Özler (2019)). For Indonesia, Cahyadi et al. (2020) find long-term effects on schooling by a cash transfer program. My model aligns with this finding, as parents increase schooling investments when receiving cash transfers.

schooling decisions, I can account for parents' responses to policy changes in the simulations and quantify the impact of nutrition diversity on child development in a low- and middle-income country context. Methodologically related to my work are the papers of Del Boca, Flinn and Wiswall (2014) and Caucutt et al. (2020), as I also explicitly model investment choices. While I use similar methods to estimate parameters, I deviate from their framework by using a different investment input (nutrition), modeling outcomes including adult skills, and using data from a lower-middle income country.

The rest of the paper is organized as follows. In Section 2, I discuss the data used and present facts on the skill gradient in Indonesia. Next, I introduce the theoretical model and describe the estimation procedure in Sections 3 and 4. In Section 6, I simulate the long-run effects of unconditional cash transfers, nutrition and schooling subsidies. I summarize remarks on results, their interpretation, and ideas for future research in section 7.

2 Data and evidence on socio-economic background and skills

I firstly describe the data used, how I construct main variables and sample selection in section 2.1. In section 2.2, I present empirical evidence for the skill gap by socioeconomic background in Indonesia. Further, I motivate model assumptions and the empirical analysis by exploring potential drivers of the skill gap using the data.

2.1 Data

I use the Indonesian Family Life Survey (IFLS) as main data source.[8] This panel survey spans from 1993 to 2014 and allows me to track children from childhood to adulthood because of low attrition rates (around 90% to 95% depending on survey wave). An additional advantage of the IFLS is the area it covers, representing 83% of the Indonesian population. The wide regional coverage provides spatial variation in food prices to exploit.[9] Further, the IFLS entails detailed information on households and their investments in children. Jointly with the available data on children's cognitive skills, this information is crucial for a model capturing the mechanisms behind the skill gap between children from different socioeconomic backgrounds.

[8] IFLS data was supplied by the RAND cooperation, for details see: Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009), Strauss, Witoelar and Sikoki (2016) and https://www.rand.org/well-being/social-and-behavioral-policy/data/FLS/IFLS.html

[9] The majority of regions not covered in the survey are in the Eastern provinces, which are very remote and poor and were not surveyed for practicality reasons.

I utilize survey waves from 1997, 2000, 2007, and 2014, excluding 1993 due to the lack of food price data. Despite the gaps between survey waves, the IFLS offers rich longitudinal data on households and their members, with interviews conducted primarily with the household head and spouse and detailed information collected on 2-3 randomly selected children per household. I select children of the household head or adopted by them. This procedure avoids including children of other parents in the household as the household head might be the decision maker and not them. I exclude children without sufficient data on investments, skills, and parental characteristics. With this procedure, the final analysis sample consists of approximately 4,563 children in early childhood, 6,329 in primary school, and 8,451 in high school. I firstly describe how I measure the main variables, before providing descriptives of the sample and empirical evidence to motivate model choices (for details on the variable construction, see Section A.1).

Cognitive skills: Cognition is measured via language, raven and numeracy tests in the IFLS. The raven test consists of matching similar shapes, while numeracy entails addition and subtraction exercises. The language test questions were drawn from the national school exam's question database. I calculate the percentage a child answers correctly of the posed exercises to get a general test score for math and for raven/language test results. Survey questionnaires for 2000-2014 are comparable, however, in 1997 language was tested instead of employing a raven test. Therefore, I group raven and language test scores in one measurement called raven test scores. All tests were designed in varying difficulty adjusted to age groups, which vary by survey round. Hence, questions vary in difficulty and content across waves. To ensure comparability I standardize scores by age. In early childhood, cognition is not measured by the survey. Therefore, I substitute skills in this period by height and weight measures, standardized using WHO Child Growth Standards (Vidmar, Cole and Pan, 2013).

Investments: For schooling investments, I use reported expenditures on education expenses as schooling fees, exam fees, uniforms, private tutoring and books in the last school year. All monetary values are calculated in nominal terms and then deflated using the Indonesian consumer price index to ensure comparability across waves. Additionally, I trim these values at 1% to avoid outliers to impact the estimation. For nutritional investments, I use the household consumption data to measure nutritional diversity following Attanasio, Meghir and Nix (2020). I calculate expenditures for five food groups: vegetables, fruits, dairy, proteins, and staples. Then, I create a variable indicating the number of food groups a child consumes (ranging from 1 to 5). I count an expenditure group if it constitute more then 5% of total expenditures for food to ensure the diversity measure represents meaningful quantities.

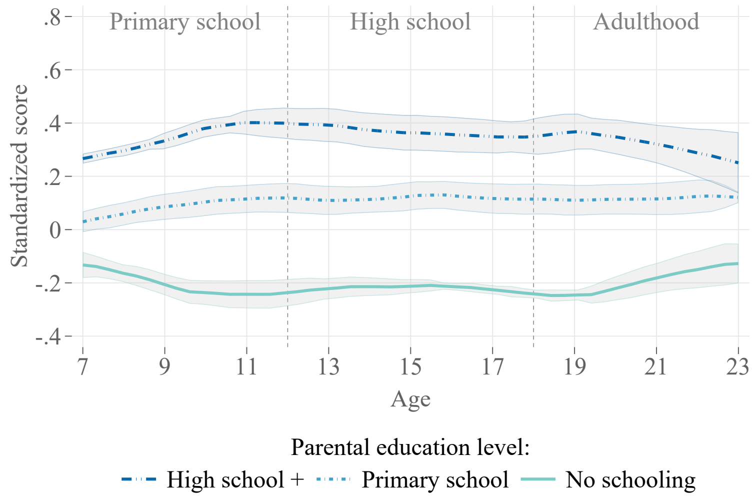
Food prices: I rely on the community questionnaire of the IFLS, which survey prices for a subset of food items in the community markets and shops. I average prices for items which correspond to the food groups described above. Consistently available across years are prices for vegetables, proteins and staples. Prices are reported as price per kilogram of the corresponding item. To convert this to a yearly price per food group for the child, I employ the following approach: first I calculate median expenditure shares of households on the food groups with available prices. Then I calculate a weighted price for each kg of nutrition bought by the household with these shares. The weights assigned are 0.14 for vegetables, 0.43 for proteins and 0.43 for staples. To scale the weighted food price by kg to a yearly price for one food group, I calculate the average kilograms consumed yearly by households using their expenditure data and food price data. To convert the average into a quantity a child consumes I use equivalence scales for Indonesia estimated by Olken (2006) for different ages and household compositions. I multiply the weighted kilogram price with this quantity equivalent to the consumption of a child per year to get a yearly price per food group.

Parental and household characteristics: For parental education, I use the parents' education level reported at the start of the child's life. I group parental education into the following levels: no schooling, primary school and high school or higher education. Other survey-reported characteristics are child gender, the number of siblings, if the household lives in a rural area, religion and child age. I calculate household income using available information on labor income and income from household-owned businesses. Assets and debts are also calculated using the reported values in the survey.

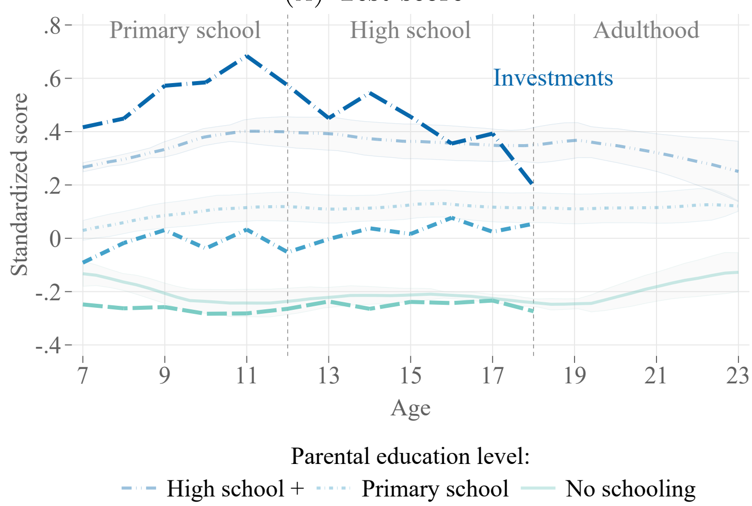
Table VII shows the sample characteristics. The final sample is gender balanced, 54% of children live in rural areas and parents on average have primary education. 88% of children declare to be Muslim. Noteworthy, a fraction of 0.34 exhibits stunting (extremely low height-for age), and a fraction of 0.09 wasting (extremely low weight-for age). These fractions highlight the food insecurity in Indonesia and importance of nutrition investments in this context.

2.2 Empirical evidence on socioeconomic background and skills

The skill gap in Indonesia is substantial and opens early in life. In Figure I, I plot averages of skills by parental education group and age for cognitive skills and in Figure VIII for height. I use parental education as indicator for socioeconomic status, as education is easier categorize than income. Households with lower educational background are mostly in the bottom of the income distribution, so education is correlated with income. Visible in Figure VIIIa, children from poorer backgrounds are smaller than their richer peers from the start of their life. These disparities continue through childhood, for both health and cognition. In adulthood, children from poorer backgrounds still have substantially lower skill levels than their peers.



(A) Test score



(B) Test score and investments

Figure I: Children skills and investments over age by parental education

Note: Skills are fitted with local mean smoothing by age and parental education groups. Parental education groups correspond to the average education of both parents. Confidence intervals displayed are at 95% level. Investments plotted are standardized schooling expenditures. Scores of skills and investments are standardized by age to have a mean of 0 and SD of 1.

Parental investments could be one driver of this disparity in outcomes. In Figure Ib, I plot standardized schooling expenditures on skill disparities to test for the existence of an investment gap in schooling (see Figure VIIIb for nutritional investments). In poorer households, investments are substantially lower throughout childhood. However, for schooling expenditures the gap widens more in primary school and closes to some extend towards the end of high school. In contrast, nutrition investment disparities are stable over childhood. Thus, parents with higher education mainly increase investments at the end of primary school, presumably for high school fees, while nutrition differences persist over time.

These investment differences can have several reasons. Foremost, parents with lower education have fewer resources to invest in their children. As shown in Table I, parents with no schooling have less income available. By that, they are more constrained in investing in children, both for nutritional investments and for schooling. Differences in investments are substantial; parents with high school education spend more than triple on education than their counterparts without schooling.

TABLE I: Potential sources for the skill gap by maternal education

	Parental education level:					
	None	Primary school	High school	F-test	Mean	Sd
Resources						
HH income	181.02	384.53	522.77	0.00	289.19	479.74
Maternal skill set						
Test score	-0.44	0.24	0.51	0.00	-0.00	1.00
Height	-0.15	0.13	0.31	0.00	0.00	1.00
Initial skill levels						
Test score	-0.23	0.21	0.37	0.00	-0.00	1.00
Height-for-age	-0.17	0.18	0.41	0.00	0.00	1.00
Childhood investments						
Food groups consumed	3.36	3.71	3.85	0.00	3.57	0.91
Education spending	2.30	5.37	7.53	0.00	5.14	10.50

Note: The last column displays p-values for the null hypothesis that means for none and high school education are equal. Skills are normalized to 0 mean, SD of 1. All values are from period 2 (age 6-11), except initial height. Income and education spending expressed in 100,000 rupees.

Apart from less available income, poorer households might be also not as effective in investing in their children. With lower levels of education or less bandwidth due to income shocks, they might not be as available to help their children with their homework for instance. Further, according to the data, they also have lower cognitive skills and worse health. This might not only result in their children having lower skills at the beginning of their life but also in investments being less effective.

Given these potential differences in effectiveness, parents with lower socioeconomic status might invest differently. To provide evidence for that, I plot the share of nutrition and schooling investments with respect to the total household income for different household income deciles in Figure II. Firstly, poorer households spend an higher share of their income on investments, particularly on nutrition. However, comparing relative shares, richer households spend a similar share on nutrition and schooling. In contrast, poorer households spend a significantly bigger share on nutrition than on schooling.

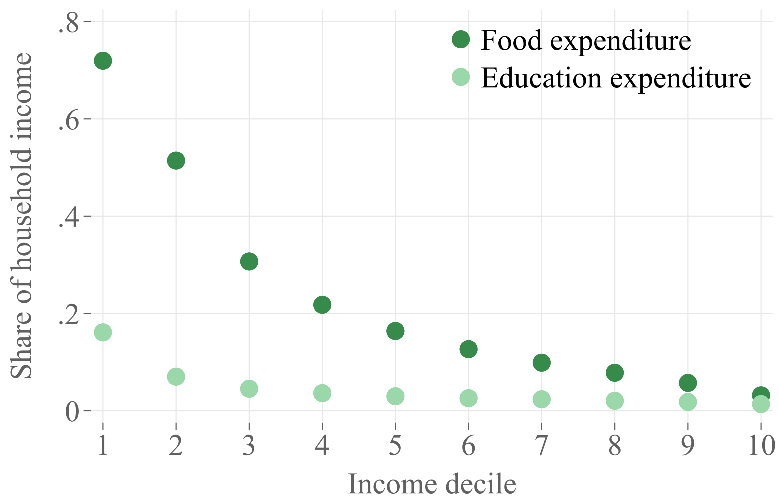


FIGURE II: Fraction of household income spend on child investments

Note: Expenditures shares are plotted as median fraction of total household income by income decile. Household income is adjusted by household size.

These different investment shares could be routed in richer households being more effective in using schooling investments and therefore, spending more on them. Nonetheless, these choices are also linked to differences in income, preferences, initial skill levels and investments prices. These underlying mechanisms complicate simulating the long-run effects of policies as nutrition and schooling subsidies or cash transfers. To be able to estimate if policies reduce the skill gap, parental responses to policies need to be simulated well as they might react to subsidies differently than to cash transfers. Additionally, their responses might vary given their socioeconomic status. For example, if parents in the bottom part of the income distribution spend significantly more money on nutrition than schooling, they might not react strongly to a schooling subsidy. Particularly, this might be the case if their investment effectiveness for schooling is low. I can only model these mechanisms in a structural model with endogenous parental investment choices. Therefore, I construct a model where parents decide on different investment inputs being constraint by income. The model also features investment productivity of households to vary by parental education. Similarly the number of siblings and if the child grows up in a rural area might influence effectiveness of investments.

However, using only for observable characteristics of the parents might miss an important feature: heterogeneity in parent types. Parents behave and think differently, leading them to make different investment decisions. For example, some parents might be better at explaining math to children than others. Therefore, their schooling investments could be more effective. Omitting these parent types might make simulations less realistic, if types do not coincide with other observable characteristics as income or education. To illustrate that they are not aligning with education and income, I plot distribution by parent's income and education groups in Figure IX. As one can see, the distribution in the lower education and income categories is skewed to the left. However, even in these categories, there is substantial heterogeneity, which can be driven by parent types, which are unobserved. Therefore, I will include parent types in the model to be able to control for these unobserved factors.

3 Model

To model skill development in a framework with endogenous parental investment decisions, I build on models by Del Boca, Flinn and Wiswall (2014) and Caucutt et al. (2020). In contrast to both, I focus on nutrition and schooling inputs instead of time inputs given the context of Indonesia where nutrition scarcity plays a dominant role. In the model, parents decide on investments into the child each childhood period (early childhood, primary school and high school). Figure III illustrates a graphic overview of the time line. Periods are determined by the child's age, following standard definitions in the literature for an early childhood period, primary education and secondary education.

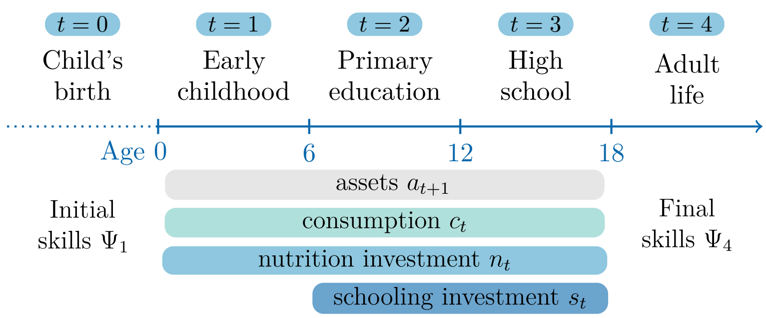


Figure III: Overview of childhood periods and parental choices in the model

In period $t = 0$, the child is born with an initial skill endowment Ψ_1. Then, in each childhood period, parents decide to invest their resources into consumption c_t, savings a_{t+1} or investments in the child I_t, deriving utility from consumption and current child skills Ψ_t. Investments I_t are nutrition investments n_t in early childhood and nutrition and schooling investments s_t in primary and high school. In the final period, the child grows up to be an adult, and no further decisions take place. Households get utility from final child skills Ψ_{T+1} and assets a_{T+1} in this period. Deriving utility from final assets ensures that parents do not deplete assets fully in the high school period to maximize utility in the last decision period. Parents are constrained by their income y_t and their decisions are influenced by the prices of investments, $p_{s,t}$ for schooling and $p_{n,t}$ for nutrition. Generally, each household is represented by a parent-child pair. Therefore, I adjust household income by household size to account for household composition (see A.1 for details). Formally, each childhood period the household maximization problem is the following:

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, n_t, s_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + p_{n,t} n_t + p_{s,t} s_t + a_{t+1} &= (1+r) a_t + y_t \\ a_{t+1} &\geq a_{min,t} \end{aligned} \tag{1}$$

Households maximize their utility U_t with respect to consumption c_t, future assets a_{t+1} and investment choices, which are nutrition n_t and an schooling s_t. All investments are associated with their corresponding prices in the budget constraint. The price for nutrition is $p_{n,t}$, and the price for one unit of schooling is $p_{s,t}$. The vector of all prices for investments is denoted by Π_t. The household cannot spend more than their current income y_t and assets a_t. Future utility depends on the evolving state space of future income and prices, as well as future household characteristics Z_{t+1} and future skills Ψ_{t+1}. Households can borrow, but not more than $a_{min,t}$, the maximum amount a household can be in debt.

The each decision period $t \in \{1, 2, 3\}$, household utility depends on consumption and skills. The utility functions take the corresponding forms:

$$u(c_t) = \ln(c_t) \tag{2}$$

$$v(\Psi_t) = \ln(\Psi_t) \tag{3}$$

In the final period $T+1$, utility exclusively depends on the final skill level of the child Ψ_{T+1} and final assets a_{T+1}:

$$V_{T+1} = u(\Psi_{T+1}) = \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \tag{4}$$

The altruism factors α_e and γ_e depend on parental education e. By this, I allow parents to value their child's skills differently depending on their education. It could be that parents with higher education, value cognitive skills more. By allowing preferences to vary by education, I can estimate if this is the case.

For parents to decide how to invest in their children, they take into account how their investments impact future skills. I assume the human capital production function to take the following form:

$$\Psi_{t+1} = \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \tag{5}$$

Future skills will depend on current investments I_t, current skills Ψ_t and a total factor productivity $\theta_t(Z_{\theta,t})$. In this context, $\delta_{1,t}$ will describe the impact investments have on future skills, which varies by period. The self-productivity of skills Ψ_t is expressed by $\delta_{2,t}$, also varying by period. By this, I ensure that the estimation is flexible enough to capture that early childhood skills might influence future skills more than skills in high school. Persistence of skills is likely to increase over childhood, and this functional form allows to capture this development flexibly. The total factor productivity is supposed to capture for instance that some parents might be able to provide a better learning environment to their children. Therefore, their investments might be more efficient and the initial skills more persistent for future skills. This productivity depends on observable characteristics $Z_{\theta,t}$, which are parental education and the age of the child. I include the age of the child to capture that children are measured at different ages, as each childhood period captures a range of ages and investments and skills might translate differently to the next period depending on the current age of the child.

Investments I_t are composed of inputs as nutrition n_t and schooling s_t investments, which are mapped to a value of total investments in the following form:

$$I_t = [n_t^{\rho_t} + a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t}]^{\frac{1}{\rho_t}} \tag{6}$$

I assume an investment function with a constant elasticity of substitution (CES) following Caucutt et al. (2020). The parameter ρ_t describes the elasticity of substitution between nutrition and schooling. The elasticity of substitution ϵ_t varies by period and can be calculated using ρ_t with $\epsilon_t = \frac{1}{1-\rho_t}$. Thus, if $\epsilon_t < 1$ investments are complements, if $\epsilon_t \geq 1$ they are substitutes. Hence, the elasticity will drive price reactions of parents. Suppose investments are substitutes and the price of one rises. In that case, it will be substituted by another investment input to some degree. If they are complements, this substitution will not happen, and overall investment might be decreased depending on the degree of complementarity. Addtionally, schooling investments have a relative productivity of $a_{s,t}$, which depends on observable characteristics. These are parental education e, age, number of siblings and unobserved parent types η. Productivity depends on parental education since one could imagine that the investments have differential effects by parents' education. Higher-educated parents might be able to buy books for schooling when the child needs them or to help the child with homework at later levels of schooling. In a similar spirit, unobserved parent type η influence productivity. Controlling for the number of siblings allows for the possibility that either siblings to help with homework or reduce the time parents can spend with the child on homework, thus reducing the productivity of schooling. I normalize the productivity of nutrition to $a_n = 1$ for identification. In early childhood $I_t = n_t$ as schooling inputs are not possible.

Depending on the productivity of each investment and the elasticity of substitution, price increases will have different impacts on investments varying by parental education and other observable factors. For instance, if food prices increase and the investments are substitutes, investments might shift to more schooling expenditure. However, if schooling investments are more productive for high-educated parents, they might have to buy less quantity to substitute for the loss in nutrition than parents with lower education. In terms of complements, the substitution would not take place. However, if schooling is more productive for high-educated parents, changes in food prices might impact them less than low-educated parents. This interplay shows why it is essential to know if investments are substitutes or complements and include productivities by socioeconomic in the assessment. This knowledge can help to design suitable policies. In the case of substitutes, a price subsidy on one product might lead to less investment in another. In case of complements, this might lead to an increase in all types of investment. Additionally, this effects might vary depending on the socioeconomic status of the parents.

To solving the model, I exploit the fact that the maximization problem can be separated into an inter-temporal and an intra-temporal problem as in Caucutt et al. (2020), Moschini (2019) and Molnar (2018). The intra-temporal problem minimizes the costs for investments for a given amount of total investments I_t. The inter-temporal problem will then maximize utility with respect to total investments and consumption. The minimization problem takes the following form:

$$\begin{aligned} &\min_{n_t, s_t} \quad p_{n,t}n_t + p_{s,t}s_t \\ &\text{s.t. } It = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{7}$$

I can derive solutions for each investment input given the total investment level. With having derived equations for the investment inputs n_t and s_t given I_t, I can reduce the maximization problem to maximizing with respect to I_t, simplifying derivations (see section A.5). Then, the inter-temporal problem can be characterized by:

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = &\max_{c_t, I_t, a_{t+1}} u(c_t) + \alpha_e v(\Psi_t) \\ &\qquad + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad & c_t + \Lambda_t I_t + a_{t+1} = (1+r)a_t + y_t \\ & a_{t+1} \geq a_{min,t} \end{aligned} \tag{8}$$

Λ_t describes the price for one unit of total investment, which arises from the results of the cost minimization (see section A.5). Given the results, investment input prices will determine the amount of each investment input and the price for one unit of total investment.

Lastly, income follows the following process:

$$\ln(y_t) = Z'_{y,t}\gamma_y + \eta'\gamma_\eta + \epsilon_{y,t} \tag{9}$$

Hence, current income depends on observable characteristics as parental education or the number of adults in the household. Additionally, income depends on parent type. The reason is, that parents which for example are very thoughtful with their children and thereby enhance their skills, might be also more successful in negotiating higher wages. Further, I assume the income shocks to be i.i.d. normally distributed to allow for income shocks. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

The model captures investment decisions in children influenced by investment prices and parental preferences, differences in investment productivities. I allow for the interplay of the budget constraint, preference parameters and productivity of skill formation differing by education and other observables as well as unobserved parent type. Doing so, I can simulate the skill gap between children from different backgrounds and their parents choices to estimate the long-run effect of childhood policies to close disparities in children's cognitive skills.

4 Estimation and calibration

To estimate the parameters of the model, I take the following steps:

1. Estimation of parent types by k-means algorithm
2. Estimation and prediction of household income by OLS
3. Estimation of skill formation parameters by joint Generalized Method of Moments (GMM)
4. Estimation of preference parameters by simulated methods of moments (SMM)

In the following paragraphs, I describe each step chronologically (for details, see appendix A.3). Firstly, I estimate the unobserved parent types. Since all model equations depend on unobserved parent type η with $k = \{1, ..., K\}$ types, these need to be estimated first. However, parent types are unobserved. To elicit them, I use the k-means algorithm in the spirit of Bonhomme, Lamadon and Manresa (2022). To determine types, the algorithm exploits the panel dimension of the data. Given the model, parent types influence income and child investments. Therefore, parents of the same type should converge over lifetime to similar values if other variations are averaged out. Hence, using the available panel data on these outcomes, one can cluster parents into types. The advantage of this method is that it allows for types whose impacts vary over childhood periods. Additionally, estimating the types outside the model is less computationally intensive, and the strategy uses empirically relevant data to determine the types.

To perform the k-means algorithm, data moments must be chosen, which are influenced by the types. In my case, these are household income, schooling expenditure and nutrition investments. I assume investments to be partly driven by unobserved parent types and that these can translate into different productivity on the labor market resulting in higher income. I calculate life-time averages of income, nutrtion and schooling investments as parent types are assumed converge over the life cycle to have the same moments values with $T \to \infty$. Thus, I can use the variation in lifetime moments in the data to determine types. To do so, the algorithm minimizes the within-cluster (type) variance. The state space is split into clusters, so that parents within a cluster are as similar as possible:

$$\min_{k \in \{1,..,K\}^N} \sum_{t=1}^{N} \sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \boldsymbol{\overline{m}_k}||^2 \tag{10}$$

where $\overline{m}_k$ is the average of the moment vector m of parent type k, t stands for time and c indexes each child the parents have. Moments are standardized to have mean zero and variance one. To run the minimization, the one needs to determine the total number of clusters K. With the help of the elbow and silhouette criteria, I determine the optimal amount of types K, as plotted in figure X. These two criteria determine the number of clusters at which variation within cluster decreases and variation between clusters increases without adding significant computing time. The optimal number is $K = 4$. Using the optimal number of clusters, I can determine for each parent pair the unobserved parent type with the algorithm. A detailed discussion of robustness checks including different number of types can be found in appendix A.3.

As second step, I estimate household income with a standard Mincer equation. Household income depends on parental education, number of household members, rurality, age of the household head, and parent types. The parameters for these characteristics will then be used to predict household income for the calibration and simulations. For these predictions, I assume the income shocks to be i.i.d. normally distributed. Thus, $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

In step 3, I estimate of the human capital and investment parameters using a joint GMM estimation. The advantage of a joint GMM estimation is that I can combine all available information to get efficient estimates. Further, I can use instruments to account for measurement error in skills. For this estimation, I solve the maximization problem of the model and derive a set of moments for the investment function parameters and human capital parameters. For the investment parameter moments, I start by deriving and rearranging the first-order conditions of the cost-minimization problem in Equation 7 to formulate the following linear relative demand equations, which I can estimate for primary and high school periods (for derivations, see A.5):

$$\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) = \frac{1}{\rho_t - 1}Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta + \epsilon_{ns,t} \tag{11}$$

The relative demand ratio between nutrition and schooling quantities will depend on observable characteristics $Z_{s,t}$. These enter the relative schooling productivity $a_{s,t}(Z_{s,t}, \eta) = exp(Z_{s,t}'\phi_s + \eta)$ following Caucutt et al. (2020). Note, as mentioned in Section 3, I normalize $a_{n,t}(Z_{n,t}) = 1, \phi_{n,t} = 0$ to identify all parameters. Thus, estimated parameters represent relative magnitudes of schooling versus nutrition productivity. The characteristics $Z_{s,t}$ include paternal and maternal education and other observable characteristics such as religion, age of the child, rural area, siblings in the household, and gender. Additionally, the productivity will depend on η, the unobserved parent type. The substitution parameter of investment inputs, ρ_t, is identified using variation in the price ratio of inputs. As schooling prices are assumed to be 1, this parameter will be identified by variation in the food price. The intuition is that if food prices increase, the reaction of parents in terms of their schooling and nutrition investments shows if they treat these inputs as substitutes or complements. If they increase schooling investments when food prices increase, the ratio would decrease and indicate inputs are substitutes. In contrast, if they decrease both inputs, they are complements. To be consistent, one assumption has to be met: food price changes need to be exogenous and not driven by parental investment decisions. Given I use variation of food prices at community level and parents investment decisions likely make a small volume of the food market, I assume them to be exogenous.

For the estimating moment conditions in a GMM framework, I need to specify instruments which are not correlated with the error term of Equation 11, such that the following assumption holds:

$$\mathrm{E}\left(\left[\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) - \frac{1}{\rho_t - 1}Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta\right] Z_{t,ns}\right) = 0 \tag{12}$$

As instruments $Z_{t,ns}$ for the GMM moments displayed in equation 11, I use observable characteristics $Z_{s,t}$, the price of inputs and parent types k. For this equation to be accurate, I need to assume that the measurement error in equation 11 is independently distributed across individuals, and no variables in the error term influence the demand ratio and instruments used for the moment equations. For this not to be true, a variable would need to influence schooling and nutrition inputs differently, as influences of the same magnitude factor out by the ratio. For example, not controlling for parent type η might bias the results as it could influence schooling differently from nutrition but be correlated with parental education. Hence, if it is in the error term, estimates are biased and Equation 12 does not hold.

I take a similar approach for the moments identifying parameters of the human capital production function. Using equation 5, I derive linear equations to estimate:

$$\frac{1}{\lambda_{ts,t+1}} S_{ts,t+1} = \phi_{\theta,t} Z_{\theta,t} + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts,t}} S_{ts} + \epsilon_{\Psi,t} \tag{13}$$

which entails as inputs Investments I_t and the a total factor productivity $\Theta(Z_{\theta,t}) = exp(\phi_{\theta,t} Z_{\theta,t})$ which can be a multiplier of inputs. S_{ts} represent measures for children's skills, namely raven and math test scores, and in early childhood height and weight. These measures represent latent cognition Ψ_t, but might be measured with measurement error and might not capture these entirely. To allow that these measures serve as a proxy for latent skills, I assume them to be linked to latent skills in a measurement system following Cunha, Heckman and Schennach (2010).The system looks like the following:

$$S_{ts_1,t} = \lambda_{ts_1,t} \ln(\Psi_t) + \epsilon_{ts_1,t} \tag{14}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t} \ln(\Psi_t) + \epsilon_{ts_2,t} \tag{15}$$

where ts stands for test scores I use in the corresponding period. Following Caucutt et al. (2020), I normalize one factor loading $\lambda_{ts_1} = 1$ each period. To identify these additional parameters, $\lambda_{ts,t}$ I exploit assumptions on covariance of these parameters and the information in the data on it (for derivations, see A.5):

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1} S_{ts_2,t+1}) S_{ts_1,t}] \tag{16}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t} S - \lambda_{ts_2,t} S_{ts_2,t}) S_{ts_1,t+1}] \tag{17}$$

As one can see, these equations use future skill measures as instruments to satisfy the orthogonality conditions. Hence, I assume that measurement error in skill measures are not correlated across periods or different measure. Left to determine are instruments for Equation

13, such that the following assumption holds:

$$\mathrm{E}\left(\left[\frac{1}{\lambda_{ts,t+1}}S_{ts,t+1}-\phi_{\theta,t}Z_{\theta,t}+\delta_{1,t}\ln(I_t)+\delta_{2,t}\frac{1}{\lambda_{ts,t}}S_{ts,t}\right]Z_{t,\Psi_t}\right)=0 \tag{18}$$

For this equation I use similar instruments as in Equation 12: observable characteristics $Z_{\theta,t}$ and current skill measures $S_{ts,t}$, as well as investment inputs n_t and s_t. To avoid that measurement error in skill measures violates the assumption, I use raven test scores as instrument for math test scores and vice versa, assuming measurement error between those measures is not correlated.

Lastly, in step 4, I estimate preference parameters γ_e, α_e and ζ. To do so, I use the optimal solution for total investments and assets (see section A.5 for details) in the simulated method of moments. I set the discount factor β to 0.98, following calibrations in the literature for Indonesia (Dutu, 2016). I match mean investments by childhood periods and parental education level and assets by period to their data counterparts (see Section A.3 for details). For the simulated method of moments and simulations, I assume household income and prices change over time. However, for simplicity, for the transition of state variables, I assume all other household characteristics to be fixed. Thus, households do not move from rural to urban areas, and the number of siblings does not change.

5 Results

I will discuss the results in order of the estimation strategy described in Section 4. Thus, I start with the parent types. These capture unobserved heterogeneity among parents, influencing their investment behavior and household income. Parent types are therefore different in investment levels and income. To get an idea of the distribution of types and differences in investments and income, I plot in Figure IV the number of observations by type, average values of income, nutrition and schooling investments for each type. The two most often occurring parent types, 0 and 1, have low income and schooling investments compared to the other types. Additionally, type 1 also has low nutrition investments. In contrast, type 2 has higher income but high education expenditure. Type 3 has high income and modestly increased investments. Types could be, in general, correlated with education. If they are correlated strongly, this will cast doubts on their identification. To check, I present the education distribution in the bottom part of Figure IV. Types are partly correlated with education, but there is still substantial variation within education groups. The share of mothers with no schooling is higher for the low-income and low-investment types 0 and 1, while the share of high school mothers is higher for types 2 and 3. The share of mothers with primary education is similar for all types. Hence, while there is some correlation between education and types, there is still some variation regarding unobserved parenting skills within education groups.

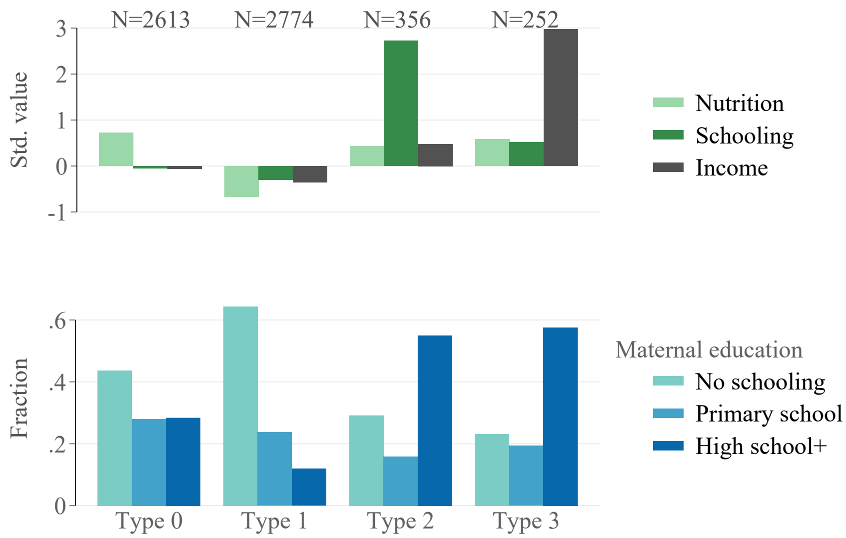


FIGURE IV: Characteristics of parenting types η (investments/resources and education)

Note: Nutrition is food groups consumed, schooling describes schooling expenditure, and income annual household income (lifetime averages by parenting pair).

Turning to the results on household income, one can observe that these parent types matter (see Table IX). Particularly, types 2 and 3 are associated with higher income. Especially type 3 has high productivity, which is the one with the highest observed income, while type 1, the lowest, is associated with a negative coefficient. In terms of magnitude, being of type 2 corresponds to an increase in household income of having a mother with a high school education. Furthermore, being of type 3 exceeds this by influencing income a third more than both parents' high school education. Unobserved parent types are likely to contribute to the gap by socioeconomic status. They are driving part of the income differences between parents. The other coefficients from the household income estimation show the expected signs and magnitudes; education and age increase income, while living in a rural area decreases it.

The GMM estimation results for investment parameters using Equation 11 reveal the degree of complementarity for investment inputs and their productivity by period (see table II and for further parameters X). Nutrition is complementary to schooling in both periods, primary and high school. Consequently, if prices for nutrition increase, parents decrease their investments in nutrition and schooling. Worth to note that the complementarity increases in high school with a higher substitution parameter ρ_t of -11.38 versus -3.75 in primary school. The complementarity is stronger than in Caucutt et al. (2020), who find for time and goods investments ranging around -1 for the US.

Table II: Estimation results for investment parameters

	Primary school		High school	
Investment elasticity:				
ρ_t	-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity	0.21		0.08	
Schooling investment productivity $\phi_{s,t}$:				
Constant	-3.68	(0.51)***	-42.17	(16.55)**
Mother primary	1.10	(0.25)***	3.06	(1.32)**
Mother high	1.87	(0.39)***	5.04	(2.15)**
Father primary	0.09	(0.16)	0.63	(0.47)
Father high	-0.08	(0.19)	0.51	(0.50)
Parenting type 1	-0.24	(0.14)*	0.06	(0.34)
Parenting type 2	4.74	(0.97)***	9.62	(4.10)**
Parenting type 3	1.64	(0.50)***	2.47	(1.29)*
Observations 27,366				

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

The high degree of complementarity in high school leads to parents responding to price changes of one input with decreasing demand for the other one stronger than in primary school. A reason for this reaction might be that in primary school, schooling is mandatory, making the demand for it less elastic. However, in high school, parents reduce investments more in their children if food prices increase as securing the households food consumption is a priority and schooling is not mandatory for the full period. For parents, it is not efficient to reallocate investments to the relatively cheaper input schooling. Reallocation does not happen because strong complementarity means that if both investment inputs increase simultaneously, this yields the highest total investment. Increasing only one is not efficient.

Considering policies, this is an essential result since decreasing nutrition prices might increase food diversity and schooling expenditure. However, this depends on how parents react to price changes (e.g., if they reallocate money to another input or spend the money for consumption). For this question, policy simulations are necessary. In general, the complementarity of schooling and nutrition is in line with findings that children's test scores increase with the availability of school meals (see Alderman and Bundy (2012), Chakraborty and Jayaraman (2019) and Aurino et al. (2020)). Sufficient nutrition increases learning ability; and further increasing both inputs yields higher skills than increasing only one.

Apart from the complementarity between inputs, schooling productivity differences might affect how parents react to price changes. Regarding productivities, Table II shows how these vary with parent type and education and Table X for other characteristics. The relative productivity of schooling increases with maternal education, especially in the last childhood period. Thus, schooling is more productive for children with mothers with high school education. Similarly, parent types 2 and 3 are more productive in schooling. Living in a rural area decreases the productivity of schooling, especially in high school. This magnitude offsets the productivity increase of having a mother with a high school education. Having siblings negatively influences schooling productivity, more so in high school, while not being Muslim increases productivity. By similar magnitude, productivity increases for female children, but only significantly in the high school. Parents with high productivity will invest a higher share in schooling than parents with lower productivity. To assess the relevance for policies, simulations are needed as the productivities impact on investment decisions in combination with changes in prices is not straightforward (see Section 6).

These parameters mentioned above describe the composition of total investments parents will supply choosing nutrition and schooling inputs. To link parental investments to skills, Table III displays estimation results from the key parameters in Equation 5. This equation quantifies the impact of parental investments and current skills on future skills. The human capital parameter δ_1 describes the impact current investments have on future skills, δ_2 characterizes the impact of current skills. They are multiplied by the total factor productivity of parents, which varies by their education and the child's age and is characterized by $\phi_{\theta,t}$.

Table III: Estimation results for human capital parameters

	Early childhood		Primary school		High school	
Human capital parameters:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

The human capital parameters, δ_1, δ_2, and the factor productivity vary by childhood period. Looking at magnitudes, investments have a higher impact early in life, with a coefficient size of 0.28, and similar impacts in primary and high school with sizes of 0.16 and 0.18. These magnitudes can be interpreted as the fraction of a standard deviation increase in test scores if investments increase by one log point. Thus, investments impact the next period's skills more in early childhood than in other periods. Looking at the impact of current skills δ_2, skill persistence increases over life. In the first period, the current skills have a lower impact on future skills (0.1 in magnitude). However, in the first period, I only used a proxy for cognitive skills, which are height and weight. These parameters are not directly comparable and only indicative in their compared magnitudes. In later periods the persistence of skills ranges around 0.2. This persistence is relatively low compared to other findings in the literature. Cunha, Heckman and Schennach (2010) find a very high persistence of cognitive skills using US data. However, in India Attanasio, Meghir and Nix (2020) find a similar low persistence for cognitive skills at age 8 as I do for Indonesia. Nonetheless, they find a higher persistence at age 12. The lower persistence in India and Indonesia compared to the US could be driven by noisier skills measures. Bailey et al. (2017) highlight that a lower persistence of cognitive skills is measured when test scores are used instead for measures of underlying intelligence, which could also explain my findings. This lower persistence could also be linked to the fade-out of some early childhood interventions.In terms of investments, I find higher impacts than Caucutt et al. (2020) and Attanasio, Meghir and Nix (2020). However, these coefficients are harder to compare due to different investment inputs and functional form assumptions.

To illustrate the magnitudes, I compute the effect of rising current skills and investments by one unit on future skills. The calculations are visualized in Figure V for each childhood period. I take average skills (1.01) and investments (3) as base comparisons for the main calculation. To illustrate what increases of one unit mean for children with low investments, I also calculate the percentage increase for base investments of one. This increase in comparison to current investments of one is higher than in the case of three, leading to a higher growth rate. This is relevant for policies, as for the same costs of one unit of investments, increasing them for the children with low investments will lead to large increases. Adding one unit of investments increases future skills by around 9% in period one and around 5% in later childhood periods. In comparison, from a lower level of investments, adding one unit induces a skill increase of 20% in the first period and around 12% afterward. In contrast, adding one unit of skills to the current skills in early childhood leads to 6% higher skills in primary skills. Later, the effect of increasing skills by one unit is higher than that of investments, increasing to around 12-15%. Thus, investing early to increase current skills in the next period leads to higher adult skills with lower costs.

Further, the total factor productivity (TFP) increases the impact skills, and investments have, as it multiplies with these values. This productivity might vary with parental education. Results in table III show that in early childhood, only parents with high school education

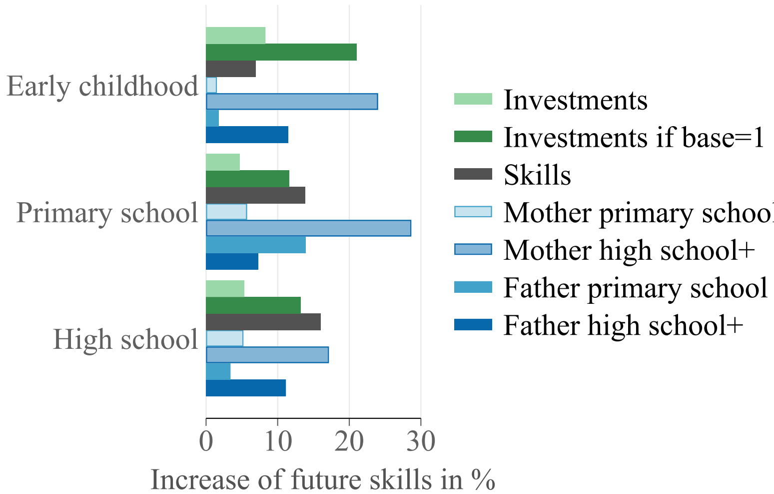


FIGURE V: Increase of future skills if characteristic/input increases by one unit

Note: Percent increase of future skills if investment or skills increase by one unit. Increases calculated with sample means as base skills (1.01) and base investments (3) if not otherwise indicated. For parental education, the base category for calculation of changes are parents with no schooling.

have a higher TFP, whereas, in later periods, also parents with primary school education do. While maternal education's impact decreases over childhood, paternal education seems to stay the same in magnitude. The impact of age is negligible. The coefficient sizes translate into percentage differences in the following period skills as depicted in Figure V. Having a mother with a high school education leads to around 25% higher next-period skills in early childhood and primary school and 18% in high school. Father's education, in contrast, has a lower impact, around 10%. These differences also magnify investment or skill input changes as they multiply with skills and investments in the skill formation equation (see Equation 5). This means, that increasing parental education can also increase the impact of child development policies significantly.

Lastly, preference parameters parents vary by education (see Table IV). Parents with higher education value cognitive skills less than their lower-educated peers compared to consumption. This is the case for the utility of current skills. Regarding future skills, parents with high education have a slightly higher valuation. In the last period, the total valuation is $\alpha_e \gamma_e$, both parameters multiplied. Given that, the valuation for skills also in the last period of childhood is higher for parents with no schooling than the ones with high school education. Thus, parents with lower education invest less in their children is not driven by their preferences. The preference for assets, ζ, after the child becomes an adult indicates that parents value assets. This parameter is not allowed to vary by education and, therefore, is the same for all groups.

Regarding their children's skills, if anything, parent's budget constraint or their produc-

Table IV: Calibrated preference parameters

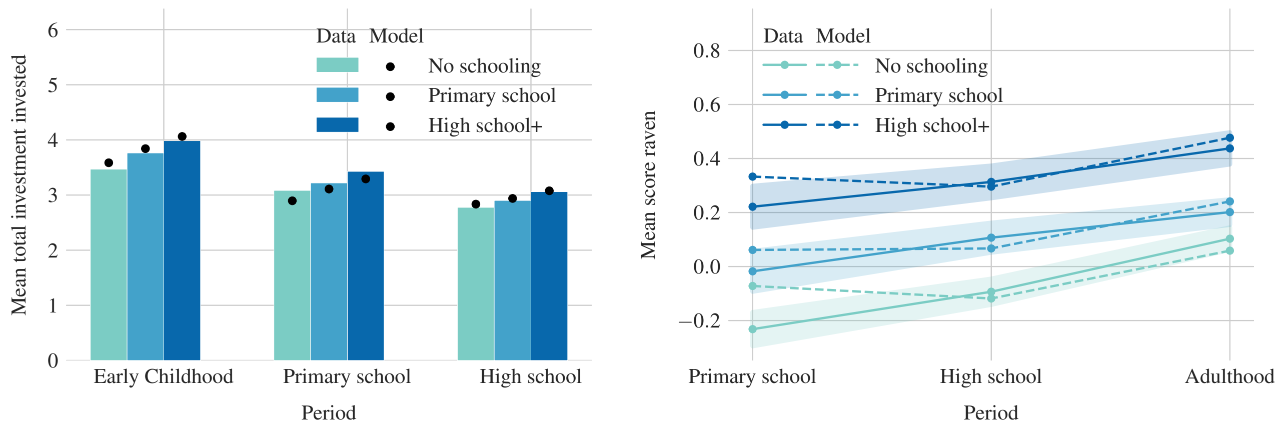
	Parental education:		
	No schooling	Primary school	High school+
For current skills:			
α_e	2.39	1.65	0.98
For final skills:			
γ_e	1.39	1.37	1.46
For final assets:			
ζ	9.99	9.99	9.99

Note: Calibration method used: simulated methods of moments. Moments targeted were investments by parental education and by childhood period.

tivities keep them from investing more in their children. These utility parameters are derived assuming that parents fully know the skill formation process. Dizon-Ross (2019) and Cunha, Elo and Culhane (2020) find that parents with lower education overestimate the impact of their skills and underestimate the persistence of current skills. Thus, they invest less than optimal in this scenario and should invest more. As I do not account for this type of imperfect knowledge in the model, the optimal value is the one observed. Hence, preference parameters are derived for these values indicating the utility derived in contrast to the one from consumption. These parents would invest more without the knowledge barrier, lowering their consumption, and the value for preferences would be even higher. Therefore, the values found here are instead the lower bound of parameters.

Regarding the model fit, I will display first the targeted moments, thus, the moments I match in the simulated methods and moments. Second, I will display untargeted moments, which are not matched in the estimation procedure. As untargeted moments, I chose the skill formation by parental education group, as these outcome and process is important for policy analysis. Comparing the targeted moments of the model with the data shows that the model does reasonably well (see Table XI). The model fits the data well regarding investments and untargeted moments for skills, as shown in Figure VI.

The model fits the data well. However, total investments in the early and primary school periods seem slightly off in the model simulations. Regarding the untargeted moments of nutrition and schooling, Figure XII shows the fit. The model fits schooling investment in primary school well and tends to simulate too high levels of schooling expenditure in high school and generally too low nutrition investments in both periods. The gap between parents of different education is fitted well, however. Looking at untargeted moments on raven test scores, I match well the horizontal gap between parents from different education backgrounds.



(A) Targeted moments: investments

(B) Untargeted moments: skills

FIGURE VI: Model fit for investment choices and skills by period and parental education

Note: Investment and skill means plotted by parental education and childhood periods.

I also fit the gap vertically well between high school and adult skills. In primary school, the levels of skills are slightly off. In Figure XI, displaying the result for math test scores, the curvature of the skill gap is better captured, but the level for low-educated parents in primary school is still off. As the model's focus is not on early childhood, I concentrate the analysis on policy experiments in primary and high school for these reasons.

I conduct two robustness checks for the model estimates. Firstly, I test if assuming food diversity to end at five does impact findings of the model by relaxing this constraint. Additionally I also test results with a different number of parent types to assess if the number of types drive results. My findings are robust to these alterations (for details, see Section A.3 and A.3).

6 Policy experiments

I simulate three common development policies, a nutrition price subsidy, a schooling price subsidy, and an unconditional cash transfer. With these policies, I target the children with parents who are in the 20% lowest part of the income distribution. I first simulate the impact of each of these policies on adult skill outcomes. Second, I simulate the impact of combining them. This means for example, allocating money to a cash transfer and one of the price subsidies. I focus on the last two periods of childhood, thus do not simulate the policies for early childhood as I do not model this period in detail. To ease the comparison of policies, I simulate them to have the same costs.

Given the same costs constraint, the cash transfer has a size of 3% of the mean average income of the lowest 20% of the income distribution. The food price subsidy is around 20%. Practically, this subsidy could be implemented using vouchers, which allow parents from the lower part of the income distribution to shop at lower prices (e.g. 20% discount). The schooling expenditure subsidy is at 99%. This high percentage means that the program pays nearly all the schooling expenditure of the household. One could treat that as a tuition waiver or scholarship. For costs, I only use the costs I can identify with my simulations. Thus, the monetary amount supplied to households is part of the program's costs but not the implementation costs. This shortcoming needs to be considered to interpret cost-effectiveness. The lack of implementation costs could be especially relevant for the last two policies, as subsidies need a distribution system of vouchers in place and shops which accept them. Further, I do not simulate any other impacts than on cognitive skills and cannot simulate general equilibrium effects. The simulations' results are displayed in Table V.

TABLE V: Policy counterfactuals - investment and skill change

	Cash transfer	Nutrition subsidy	Schooling subsidy
Change in mean adult skills (SD):			
All targeted	0.00	0.04	0.03
Change in mean investments (%):			
Investments	1.65	16.29	8.87
Nutrition	1.57	15.92	6.80
Schooling	1.46	18.44	90.54
Costs in 100,000 rupees per child:			
Per 0.01 SD increase	1676.02	210.28	288.96
Total amount	7.60	7.60	7.60

Note: Policies are designed to have the same costs (in 100,000 rupees $\sim$ \$7), resulting in a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

Unconditional cash transfers have little impact, supporting the conclusion of limited effects of cash transfers on cognitive skills summarized by Molina Millán et al. (2019) and Baird, McIntosh and Özler (2019). A food price subsidy is most effective for the same costs, with an average increase in adult skills of 0.04 SD. A school price subsidy is slightly less effective than a food subsidy, with an increase of 0.03 SD. This result reflects that it is cost-effective to target parental investment behavior via price incentives. By decreasing one input price, both inputs increase in quantity. This behavior is a direct consequence of the complementarity of nutrition and schooling expenditure. The increase in investments is higher than in the case of unconditional cash transfers. Therefore, skill outcomes increase. This increase in food diversity with price subsidies complements findings of Kaul (2018) and Krishnamurthy, Pathania and Tandon (2017). These evaluations find a price subsidy in India to increase households food diversity. In contrast Jensen and Miller (2018) do not find any increases in nutrition diversity for a staple subsidy in China. Apart, the evidence on school meals supports my findings. Provision of school meals has been found to increase cognitive skills in several context (see Alderman and Bundy (2012), Frisvold (2015), Chakraborty and Jayaraman (2019) and Aurino et al. (2020)). Additionally, if the healthiness of school meals increases, they yield higher impacts, as found in an intervention in the United Kingdom (Belot and James, 2011). Extending these findings, I further find parents to increase also schooling expenditure, which additionally increases child outcomes.

A detail to note is that total investments into schooling increase little in the schooling subsidy scenario compared to the food subsidy. This behavior is partly driven by period effects. It is most effective for parents to increase investments in high school and less in primary school (see Table XIII). In contrast, with the food subsidy, parents increase mean investments in both periods. The increase in skills in the high school period translates into adult skills with more persistence than in primary school. Therefore, the schooling subsidy is nearly as effective as nutrition, even if investment levels change less on average. In general, the high degree of complementarity between nutrition and schooling investments leads to strong reactions of parents to price changes.

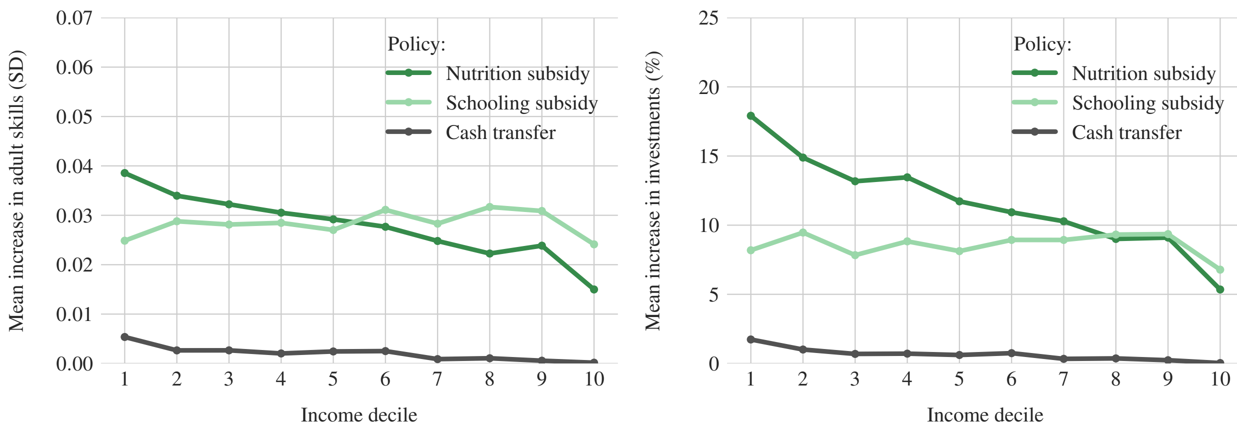
TABLE VI: Policy combination counterfactuals - investment and skill change

	Cash+ nutrition	Cash+ schooling	Nutrition+ schooling	Nutrition subsidy
Change in mean adult skills (SD):				
All targeted	0.04	0.03	0.06	0.10
Change in mean investments (%):				
Investments	17.55	10.51	26.49	48.17
Nutrition	17.09	8.37	23.94	47.26
Schooling	20.16	93.30	131.66	63.61
Costs in 100,000 rupees per child:				
Per 0.01 SD increase	387.52	483.49	267.80	157.45
Total amount	15.25	15.31	17.31	15.25

Note: Costs are expressed in 100,000 rupees ($\sim$ \$7), combined policies are a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy. The nutrition subsidy is 51% to be cost-equivalent to the cheapest combination.

Combining the policies shows that the interventions have no additional increase in skills when jointly implemented (see Table VI). Hence, there are no significant dynamic complementarities between these two policies when one considers parental responses. However, parents increase their investments, which leads to bigger costs. The increase in skills is effectively lower though, which which is why jointly implemented policies are not cost-effective even if they maximize impact on investments. It is more cost-effective to implement the nutrition subsidy alone.

As these policies are targeted toward the lowest 20% of the income distribution, I now extend the analysis to the entire population to see if there are differential effects by socioeconomic status. To do so, I simulate the described policies for the full sample, and then plot mean effects by income decile (see Figure VII). Overall, I find that nutrition subsidies and cash transfer impacts decrease with income. In contrast, schooling subsidy effects slightly increase. In support of the stronger impact of nutrition subsidies on children from poorer households, Aurino et al. (2020) find poorer children to significantly stronger profit from the proposition of school meals in Ghana.



(A) Increase in skills (SD)

(B) Increase in investments (%)

FIGURE VII: Policy impacts by income decile

Note: Plotted are mean increases in cognitive skills and investment changes in percent from baseline by income decile for each policy.

Nutrition price subsidies incentivize parents in the lower part of the income distribution to invest more in nutrition. In contrast, parents in the upper part of the distribution react to a lesser extent in increasing their investments. The opposite is true for schooling subsidies. Parents in the lower part of the income distribution are more effective at producing investments with increased nutrition investments and less effective regarding schooling. Consequently, they spend a higher share of investments on nutrition which leads to them reacting stronger to nutrition price changes. Hence, a schooling price reduction has smaller effects on children in this part of the income distribution. Additionally, one can observe that unconditional cash transfers mainly increase investments for the lowest part of the income distribution, while later, parents react only marginally in their investments. This pattern indicates that cash transfers can help lift the budget constraint of the ultra-poor. The top parts of the income distribution are not as budget-constrained leading to negligible effects on cognitive skills.

Regarding cost-effectiveness, nutrition subsidies still outperform other policies (see Table XIV). Given the differential reaction of parents by socioeconomic status, nutrition subsidies reduce inequality in skills most.

7 Concluding remarks

In this paper I build a model for child development with endogenous parental investment decisions to simulate the long-run effectiveness of development policies to reduce inequality in cognitive skills by socioeconomic background. The model entails parents investment decisions on nutrition diversity and education expenditures to allow simulations to take into account parent's responses to the policies. I estimate the model with Indonesian panel data. My findings show that subsidizing investment prices is more effective than cash transfers at same costs. A nutrition price subsidy targeted to parents in the lowest 20% of the income distribution increases adult skills by 0.04 SD, and a schooling subsidy by 0.03 SD. In contrast, cash transfers have a negligible impact on cognitive skills. However, they support the most income-constrained parents in investing more in their children. Combining different policies is not cost-effective, as instead increasing the nutrition subsidy alone yields higher effect sizes.

I also test if the effect of policies vary along the income distribution as parents might react differently to price changes or more income via cash transfers. My findings suggest that parent's respond differently by socioeconomic status to these policies impacting effect sizes. Nutrition subsidies increases skills most for the bottom part of the distribution reducing inequality, while the effect decreases for children from richer backgrounds. Similarly, the effects of cash transfers, albeit already small, decline further with income. For the upper part of the income distribution, the effect of subsidizing schooling is higher than the impact of nutrition subsidies.

Nutrition subsidies are effective for children from poorer backgrounds as their parents spend a large share of their income on nutrition. They do as they are less effective in using schooling investments effectively than their richer peers. These productivity differences are mainly driven by parental education and living in a rural area, as well as unobserved parent types. Given the larger share of income poorer parents spend on nutrition, they react strongly to food price changes, and hence, to the decrease of prices of a food subsidy. Consequently, parents increase both investment inputs, nutrition and education expenditures, with a food price decrease as inputs are complementary. Therefore, overall investments increase and with them also future skills.

Future research could focus on extend the model used by accounting for information disparities between parents from different socioeconomic statuses to investigate how they influence parents' responses to policies. Recent work by Dizon-Ross (2019) and Cunha, Elo and Culhane (2020) shows that parents with lower levels of education overestimate their children's skills and the impact of their investments compared to their peers. They also tend to underestimate the importance of early life investments driven by the persistence of current skills. Closing these information differences could lead to lower inequality. Further, modeling intra-household allocation among siblings and the effects older siblings have on the development of cognitive skills of younger ones could give additional insights for policies to reduce inequality. Calvi (2020) and Brown, Calvi and Penglase (2021) find household poverty to be shared unequally between household members. Knowing if and which children of the household are most impacted by this and in which setting could have implications for the targeting of policies. With richer data on all household members, dynamics might be uncovered. These dynamics could also play a role in the analysis and targeting of policies.

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A Appendix

A.1 Data

Food prices and nutrition investments

To capture nutritional diversity, nutrition investments are proxied by the number of food groups. Food groups in the consumption data are staples, protein, dairy, vegetables, and fruits. If the household expenditure on one food group is more than 5% of the total expenditure, I count it an investment in this food group. Due to data constraints, I cannot identify if household consumption aligns with the child's nutrition. However, I assume that it is a good enough proxy for nutritional diversity as it is unlikely that children receive entirely different food than the one bought by the household. Nutrition diversity is expressed by a measure between 1 and 5, with $n_t = 5$ meaning that a child consumes all five food groups and $n_t = 1$ that it consumes only one food group.

For food prices, I rely on the community surveys in the IFLS, which surveys food prices in the community markets and shops. I construct unit prices of protein, staples, and vegetables, which are the most prominent consumption expenditure groups and have the most reliable price data (in terms of units). Then I build the food price by weighting prices by the median consumption fraction for households in the sample consuming all three groups. This leads to a weight of 0.43 for staples, 0.14 for vegetables, and 0.43 for meat. These prices are then scaled by the average kilograms consumed by households using equivalence scales for Indonesia estimated by Olken (2006) for different ages and household compositions. I use these equivalence scales and median prices to calculate the median amount of kg consumed by a household. This amount I then multiply by the factor an additional child of the corresponding age from the household equivalence scale and the median regional food price mentioned above.

Household income and assets

I sum all income reported for the household. This includes business and farm business income, as well as all other income received by any of the household members. Further, this entails non-labor income, the number of transfers, retirement payments, and scholarships received. I adjust household income by the household size for the calibration. For that, I use Olken (2005) equivalence scales derived for Indonesia. As these are derived from aid allocated by the Raskin rice program to different family structures, I assume they will mimic the family's income and how it translates into consumption. Deaton and Zaidi (2002) and Batana, Bussolo and Cockburn (2013) state that the wildly used modified OECD scale or square root scales suit high-income countries. Using the scale for low-income countries might overestimate the degree of the economics of scale, as durables are easier to share than food, a significant fraction of the expenditure in low-income countries. Further, they tend to overestimate the cost of children. Hence, I use the scale estimated by Olken (2005), which is higher. Thus the economics of scale are lower. To compare the differences, I convert the scales in the following structure:

$$N^{eq} = (n_a + \alpha n_c)^{\theta} \tag{19}$$

where n_a is the number of adults in the household, and n_c is the number of children. α is the cost of children, and θ expresses the economies of scale. In the square root scale, $\alpha = 1$ and $\theta = 0.5$. In contrast, Olken (2005) estimate $\alpha = 0.93$ and $\theta = 0.85$, which confirms Deaton and Zaidi (2002)'s concern that the economies of scale are lower, thus θ higher in low-income countries. This also goes with Santaeulàlia-Llopis and Zheng (2017), who estimate scale parameters in Malawi to be higher than the OECD ones.

For assets, I sum all assets reported in the data, which are expressed in monetary value. This entails real estate owned, land, livestock, machinery, household appliances, savings, jewelry and furniture. I subtract from assets the reported amount of debt of the households. Then I adjust the assets with the household equivalence scale.

Skill measures

For cognitive skills outcomes, cognitive tests conducted by the survey team are available, which I standardize by age. The IFLS has several test score metrics available: In 1997, a math test with 40 questions was conducted for the following age groups: 7-9, 10-12, and 13-24, and the same was done for a language evaluation. For younger ages, no test scores are available. Therefore, in the early childhood period, only health outcomes can serve as a measure of skills. For 2000, 2007, and 2014 a raven test was conducted with 12 questions, followed by a math test of 5 questions. These were designed in 2 versions, one for age group

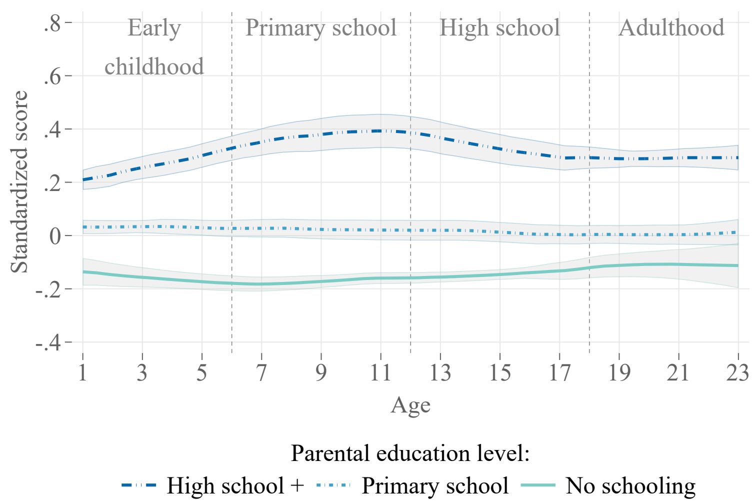
7 to 14, the other 15 to 24. In both cases, the number of correct answers is standardized by age and year. Adult respondents answered a cognitive test in 2007 and 2014. The tests ask them to remember ten words for a short period, and a second round asks how many they remember after some minutes. In 2014 additionally, a simple subtraction exercise was asked. Adult test scores are standardized by year to avoid some candidates being counted double. As cognitive measures during childhood, raven or language and math scores are taken, while for adults, a word- and math tests are used.

A.2 Stylized facts and descriptives

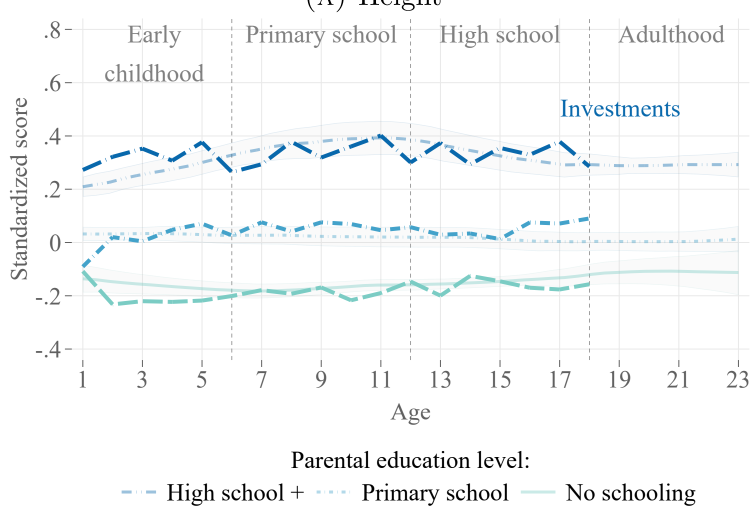
TABLE VII: Sample characteristics

	Mean	SD	Min	Max
Female	0.50	0.50	0	1
Rural	0.54	0.50	0	1
Islam	0.88	0.33	0	1
Mother's years of education	5.50	4.12	0	18
Father's years of education	6.58	4.38	0	18.5
Birth year	1990.88	6.53	1979	2007
Household income	270.65	331.2	0	3982.9
Weight-by-age	-1.16	1.44	-4.99	4.92
Height-by-age	-1.49	1.27	-4.98	4.97
Stunting	0.34	0.47	0	1
Wasting	0.09	0.28	0	1
Mother's age	41.30	9.15	17	78
Father's age	46.84	10.5	20	96
Adult household members	3.93	1.82	0	8
Household members <18	1.86	1.36	0	5
Observations	19,343			

Note: Monetary values are deflated and reported in 100,000 Rupees.



(A) Height



(B) Height and investments

FIGURE VIII: Children skills and investments over age by parental education

Note: Skills are fitted with local mean smoothing by age and parental education groups. Parental education groups correspond to the average education of both parents. Confidence intervals displayed are at 95% level. Investments plotted are standardized nutrition investments. Scores of skills and investments are standardized by age to have a mean of 0 and SD of 1.

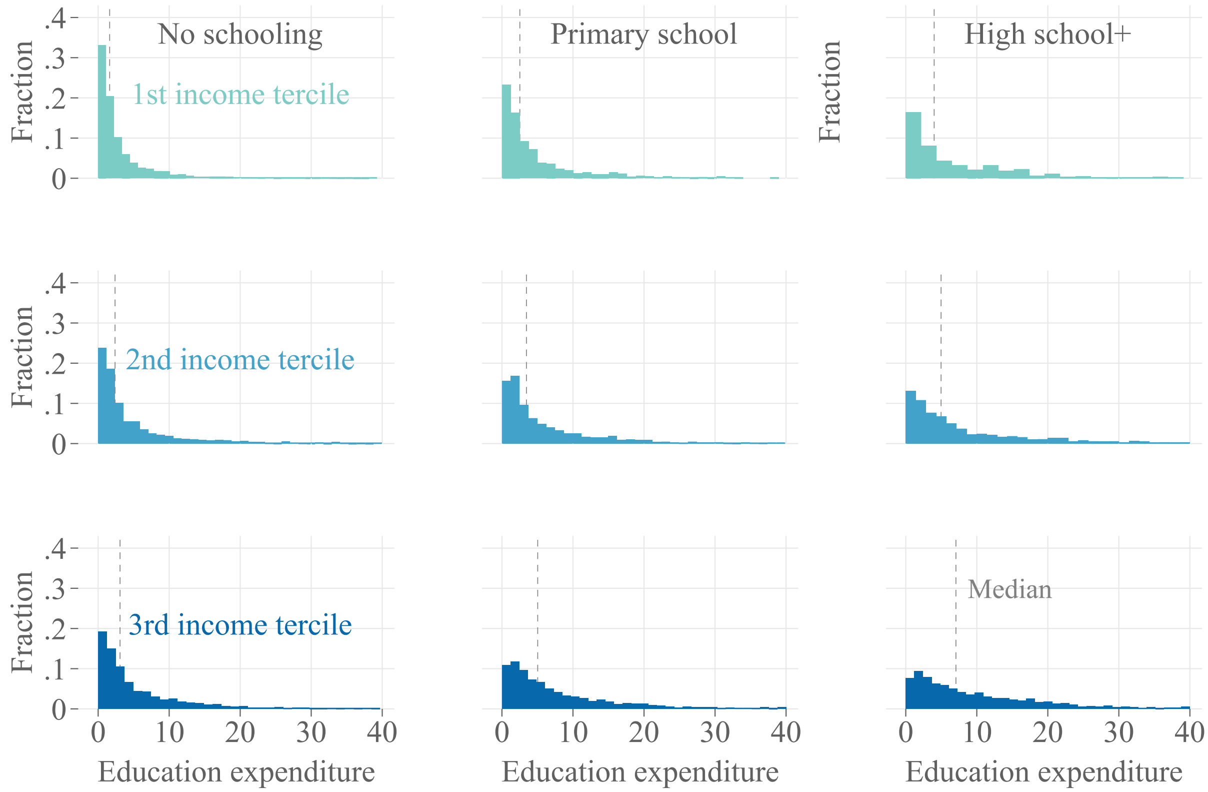


Figure IX: Heterogeneity in education spending by parental background

Note: Education spending histograms by parental education level and household income (in terciles). Parental education groups correspond to the average education of both parents. Expenditures are expressed in 100,000 rupees. The grey-dashed line indicates the median value for that category.

Table VIII: Sample characteristics by period

	Early childhood	Primary school	High school
Food groups	3.67	3.61	3.58
Schooling spending	0.24	2.61	6.00
Age	3.02	8.84	15.34
In school	0.06	0.93	0.73
Observations	4,563	6,329	8,451

Note: Monetary values are deflated and reported in 100,000 Rupees.

A.3 Estimation and calibration details

K-means algorithm

I follow Bonhomme, Lamadon and Manresa (2022) to estimate the unobserved types of parenting skills outside of the model. To do so, I build means over the life-cycle of schooling, nutrition investments, and household income for each parent couple. I then standardize these and run the k-means clustering procedure, which will allocate each household to the cluster whose moments have the least distance to the cluster mean.

To estimate heterogeneity groups using the k-means clustering algorithm, I need to choose the number of clustering groups K. As this is a data-driven approach, they are not known before but data can be used to determine them. To do so, I use the commonly used Elbow statistic. For a given number of clusters K, the algorithm minimizes the total within-cluster variance:

$$\min_{k\in\{1,..,K\}^N} \sum_{t=1}^{N}\sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \boldsymbol{\overline{m}_k}||^2 = SSE_k \tag{20}$$

To compare Elbow statistics, the variance SSE_k is calculated for each number of clusters run, $k = 1; ...; K_{max}$. These statistics are then plotted against their corresponding number of clusters, as seen in figure Xa. With an increasing number of clusters, the variance decreases as observations within a cluster become more similar. The optimal number of clusters is at the kink in the plot, the point where the decrease in SSE changes the most. Adding more clusters than at this kink would have limited value in explaining the variation in the data. Another commonly used measure is the silhouette criterion in figure Xb. The higher the criteria value, the more the two clusters are different from each other. Thus, the borders between them are well defined.

As shown in figure Xa, the Elbow criteria determines the optimal amount of clusters K to be 4. The silhouette criterion is maximized at two but also high at 4. To check if the number of clusters drives the results, I run the GMM estimation for $K \in \{2, 3, 4, 5\}$ clusters. As one can see the results for $K = 2$ in table XV, $K = 3$ in table XVI, $K = 5$ in table XVII are comparable to the main results in table X with $K = 4$. Coefficients and standard errors only vary marginally. Thus, the amount of clusters does not drive the results and, if anything, adds explanatory power. More clusters seem to explain more unobserved heterogeneity in investments, as schooling productivity varies by type. However, after $K = 4$, the amount of observations decreases by type, as shown in table XII. Hence, increasing the computational burden further has little reward. This is confirmed by the fact that these amounts exceed the amount determined to be optimal by the elbow criterion.

Household income

To estimate household income, I regress parental education, number of household members (adults and children), rurality and age of the household head, and parenting skills on household income. Additionally, I include year and province fixed effects. Thus:

$$\ln(y_t) = Z'_{y,t}\gamma_y + \eta'\gamma_\eta + \epsilon_{y,t} \tag{21}$$

Here, $Z_{y,t}$ are the named household characteristics that can vary by period. η are the unobserved parenting skills I assume to influence household income, as it is likely that characteristics resulting in productive parents also translate at least partly into higher wages. Results can be found in table IX. I use the resulting coefficients to predict future household income for the calibrations and simulations. Further, I assume the income shocks to be i.i.d. normally distributed. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

Transition of other household characteristics

I assume all household characteristics to be stable over time, except the year, age, and age of the household head. The households observed at start of their life are either recorded firstly in 1997 or 2000. For the transition to the next period, I get either 2000 or 2007 for 1997 or 2007 for 2000 (observed for the first period, as I know next period). Afterwards, due to the survey design, all future waves are seven years apart. Thus, I apply that to simulate the year in which the child is observed in the next period. Then, I apply this gap to its age and the household heads age. Knowing the next year then allows me to allocate the correct food price for the given community in that year to the simulated period. Thus, I assume households do not move. Further, I assume the number of household members and other children in the household to be stable across childhood, the same for the location in a rural or urban area.

Skill formation estimation

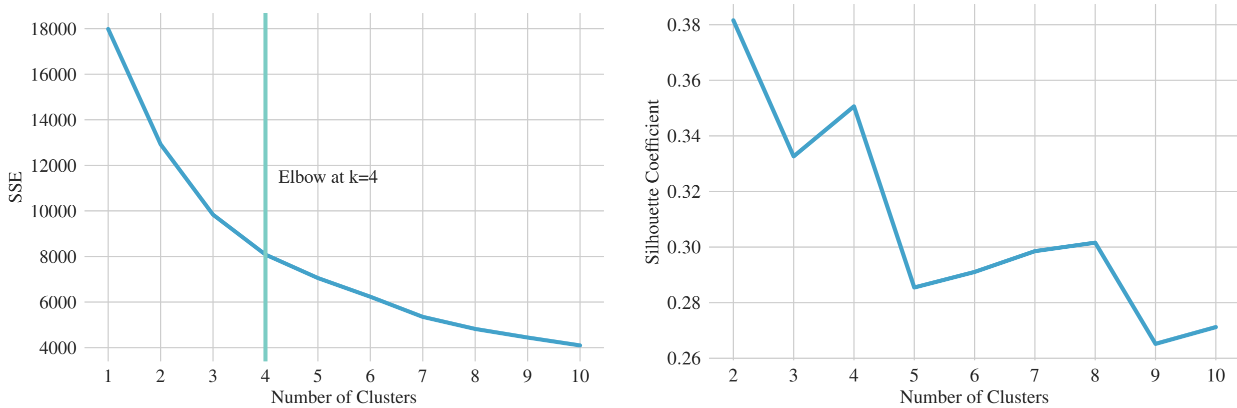
Regarding the GMM estimation, two obstacles driven by data constraints occur. Firstly, only nutrition inputs are available to measure investments in the first period. Thus, there is no stage with relative investment input ratios, which can then be plugged into the human capital parameters. Hence the food groups are directly plugged into this equation. Further, I do not observe cognitive skills in the early childhood. Hence, I use height and weight as a proxy. Therefore, $\delta_{2,1}$, the persistence of skills cannot be directly compared to the parameters in later periods, as it measures the persistence of height and weight on future cognitive skills.

Second, I assume nutrition is unconstrained, however I only observe food groups up to five. Therefore, I conduct robustness checks in case it is constrained to 5. If nutrition is constrained, the optimal demand ratios for the GMM moments hold only if $n_t < 5$ (see A.5 for details for $n_t = 5$). In the main specifications, I also include $n_t = 5$, assuming that it does not drive the results. As a robustness check, I dropped them and ran the results without using observations with $n_t = 5$ to estimate the relative demand equations (see Table XVIII). The results are relatively similar, which indicates that this subgroup does not drive the general results. If anything, the estimates are less precise, but this could also come from the smaller sample. However, dropping them introduces selection. Thus the results have to be taken with a grain of salt. Future work should exploit how these constraints bias the estimation results. For the calibration, I calibrate the model with and without the constraint without assets and do not see substantial differences. As with assets the constraint induces complex solutions, I then proceed without constraint, assuming that I observe only up to 5 food groups which can translate into 5 or more as investment in reality.

Calibration

To calibrate the model, I use the optimal solution for investments and assets derived section A.5. I match model and data investment means by parental education and childhood period and assets by childhood period to get γ_e and α_e and ζ. To calibrate the model, I use the data from period one and simulate periods two to four with it, to then compare it to the data I observe in those periods in the survey. For a_{min}, the maximum amount households can borrow, I use the average debt I observe in the data in a given year.

A.4 Estimation results



(A) Elbow criterion

(B) Silhouette criterion

FIGURE X: Criterion plots to determine number of clusters for parenting skills

Note: K-means algorithm run for different number of clusters to determine correct number for the following estimation. Plotted are on the right-hand side the within cluster variance, on the left-hand side the Silhouette coefficient by number of clusters used.

TABLE IX: Estimation results for household income

	Log(income)	
Father primary education	0.152***	(0.014)
Father high school+	0.422***	(0.016)
Mother primary education	0.112***	(0.014)
Mother high school+	0.294***	(0.017)
Parenting type 1	-0.375***	(0.012)
Parenting type 2	0.671***	(0.028)
Parenting type 3	1.441***	(0.027)
Father age	0.053***	(0.003)
Father age squared	-0.001***	(0.000)
Rural area	-0.348***	(0.012)
Adult household members	0.104***	(0.003)
Non-adult household members	0.016***	(0.004)
Constant	2.733***	(0.079)
Year fixed effects	Yes	
Province fixed effects	Yes	
Observations	36,169	

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1.

Table X: Estimation results for skill formation parameters

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity			0.21		0.08	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.51)***	-42.17	(16.55)**
Mother primary			1.10	(0.25)***	3.06	(1.32)**
Mother high			1.87	(0.39)***	5.04	(2.15)**
Father primary			0.09	(0.16)	0.63	(0.47)
Father high			-0.08	(0.19)	0.51	(0.50)
Age			-0.05	(0.04)	3.14	(1.30)**
Female			0.05	(0.13)	1.29	(0.61)**
Rural area			-2.64	(0.53)***	-5.19	(2.22)**
No. of siblings			-0.73	(0.14)***	-2.14	(0.88)**
Mother not Islam			0.39	(0.22)*	1.68	(0.85)**
Parenting type 1			-0.24	(0.14)*	0.06	(0.34)
Parenting type 2			4.74	(0.97)***	9.62	(4.10)**
Parenting type 3			1.64	(0.50)***	2.47	(1.29)*
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

Table XI: Model fit - targeted moments

	Model	Data	SD	Difference
No schooling:				
Early childhood	3.60	3.47	0.80	0.16
Primary school	2.90	3.08	0.94	-0.20
High school	2.82	2.78	1.13	0.04
Primary school:				
Early childhood	3.87	3.76	0.83	0.13
Primary school	3.13	3.22	0.99	-0.09
High school	2.94	2.90	1.16	0.04
High school+:				
Early childhood	4.06	3.98	0.80	0.10
Primary school	3.29	3.43	1.08	-0.13
High school	3.08	3.06	1.26	0.01
Assets:				
Early childhood	620.00	763.38	829.21	-0.17
Primary school	818.75	937.98	1045.17	-0.11
High school	1222.53	1128.23	1172.96	0.08

Note: Calibration method used: simulated methods of moments. Differences are expressed in standard deviations. Values are total investments by parental education and childhood period and for assets by period.

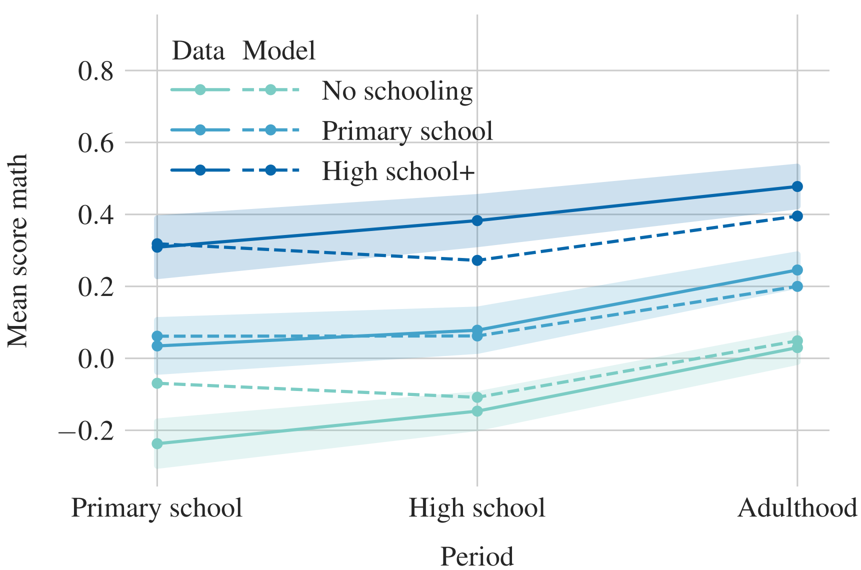
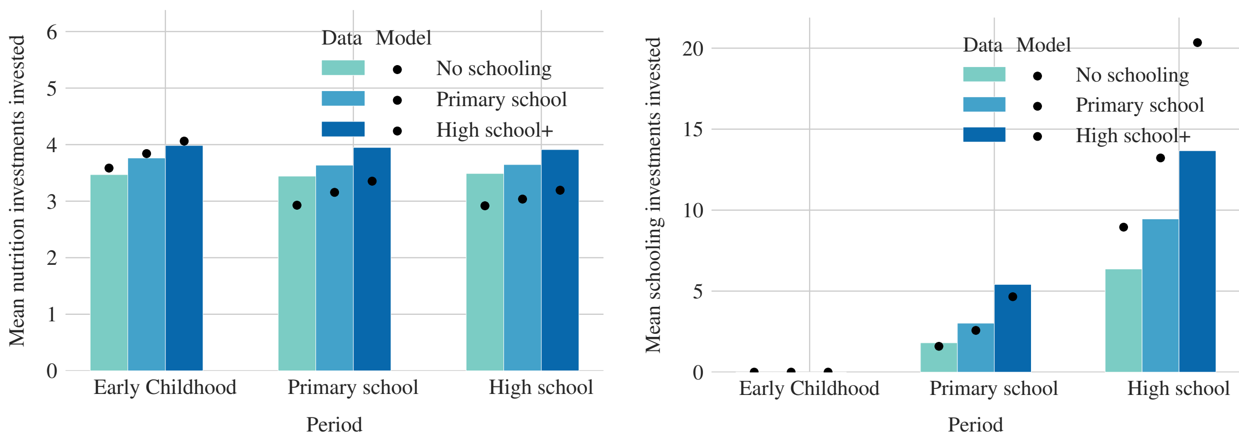


Figure XI: Model fit for untargeted children's skills by period



(A) Nutrition

(B) Schooling expenditure

FIGURE XII: Untargeted moments for investment input choices by period

Note: Investment inputs means plotted by parental education and childhood periods. Black dots are corresponding simulated moments.

A.5 Derivation Formulas

Inter-temporal solution n_t and s_t and relative demands

To derive the relative demands we take first-order conditions for the minimization problem:

$$\begin{aligned}&\min_{n_t,s_t} \quad p_{n,t}n_t + p_{s,t}s_t \\ &\text{s.t. } It = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}\end{aligned} \tag{22}$$

The Lagrangian looks the following:

$$\mathcal{L} = p_{n,t}n_t + p_{s,t}s_t - \lambda_{1,t}(I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}) \tag{23}$$

Deriving first order conditions in period 2 and 3:

$$\frac{\partial \mathcal{L}}{\partial s_t} = p_{s,t} - \lambda_{1,t}(a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = p_{n,t} - \lambda_{1,t}(n_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{1,t}} = I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} = 0 \tag{26}$$

Taking ratios $\frac{\frac{\partial \mathcal{L}}{\partial n_t}}{\frac{\partial \mathcal{L}}{\partial s_t}}$ leads:

$$\frac{p_{n,t}}{p_{s,t}} = \frac{n_t^{\rho_t-1}}{a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t-1}} \tag{27}$$

which allows to get n_t in terms of s_t:

$$n_t = \left(\frac{p_{n,t}}{p_{s,t}}a_{s,t}(Z_{s,t},\eta)\right)^{\frac{1}{\rho_t-1}} s_t = \Phi_1 s_t \tag{28}$$

and vice versa:

$$s_t = \Phi_1^{-1} n_t \tag{29}$$

Price for total investments Λ_t and relative demands I_t and I_{t+1}

The price for total investments I_t is supposed to mimic the cost for one unit of investment, thus:

$$
\begin{aligned}
E_t &= \Lambda_t I_t \\
\Lambda_t &= \frac{E_t}{I_t} \\
\Lambda_t &= \frac{p_{n,t} n_t + p_{s,t} s_t}{[a_{s,t}(Z_{s,t},\eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}}
\end{aligned} \tag{30}
$$

To calculate prices we use 28 to get expressions for n_t in terms of s_t:

$$
n_t = \left(\frac{p_{n,t}}{p_{s,t}} a_{s,t}(Z_{s,t},\eta) \right)^{\frac{1}{\rho_t - 1}} s_t = \Phi_1 s_t \tag{31}
$$

Replacing n_t in yields in 30 with moving s_t out of E_t:

$$
\begin{aligned}
\Lambda_t &= \frac{s_t(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t},\eta) s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}}} \\
&= \frac{(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t},\eta) + \Phi_1^{\rho_t}]^{\frac{1}{\rho_t}}}
\end{aligned} \tag{32}
$$

Intra-temporal solution for I_t

We can use the total price of investment equation 30 for the maximization problem to derive solutions for I_t, c_t and a_{t+1} :

$$
\begin{aligned}
V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, I_t, a_{t+1}} & \; u(c_t) + \alpha_e v(\Psi_t) \\
& + \beta_t V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\
\text{s.t.} \quad c_t + \Lambda_t I_t + a_{t+1} &= (1+r)a_t + y_t \\
a_{t+1} &\geq a_{min,t} \\
\text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \\
V_{T+1}(\Psi_{T+1}) &= \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\
u(c_t) &= \ln(c_t) \\
v(\Psi_t) &= \ln(\Psi_t)
\end{aligned} \tag{33}
$$

Which gives the Lagrangian:

$$\mathcal{L} = u(c_t) + \alpha_e v(\Psi_t) + \beta_t V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ - \lambda_t(c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t) - \xi_t(a_{min,t} - a_{t+1}) \quad (34)$$

T=3 here, because the period 3 is the last one, where the household makes decisions. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial I_t} = \beta_t \frac{\partial V_{t+1}}{\partial I_t} - \lambda_t \Lambda_t = 0 \quad (35)$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = u'(c_t) - \lambda_t = 0 \quad (36)$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \xi_t + \mathbb{1}\{t < T\}(\lambda_{t+1}\beta_{t+1}(1+r)) + \mathbb{1}\{t = T\}\beta_t \frac{\partial V_{T+1}}{\partial a_{T+1}} \quad (37)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t = 0 \quad (38)$$

$$\frac{\partial \mathcal{L}}{\partial \xi_t} = a_{min,t} - a_{t+1} = 0 \quad (39)$$

$$(40)$$

Following these one can derive a solution for I_t. First one needs to derive after I_t, which will vary by period due to the continuation value. In period 3, the continuation value looks the following:

$$\begin{aligned} \beta_t V_{T+1}(\Psi_{T+1}) =& \beta_t(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \\ \text{with } \Psi_{t+1} =& \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \end{aligned} \quad (41)$$

Plugging it in V_{t+1}:

$$\beta_t V_{t+1}(\Psi_{t+1}) = \beta_t(\alpha_e \gamma_e \ln(\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}}) + \zeta \ln(a_{T+1})) \quad (42)$$

Thus:

$$\beta_t \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t \delta_{1,t} \alpha_e \gamma_e}{I_t} = \frac{K_t}{I_t} \quad (43)$$

For period 2:

$$\beta_t V_{t+1}(\Psi_{t+1}) = \beta_t(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta_{t+1}\beta_t(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \quad (44)$$

which is:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t}))I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}}) \\ & + \beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}\Psi_{t+1}^{\delta_{2,t+1}}) + \zeta \ln(a_{t+2}))\end{aligned} \tag{45}$$

plugging in Ψ_{t+1}:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})) \\ & + \beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}}) \\ & + \zeta \ln(a_{t+2}))\end{aligned} \tag{46}$$

Thus:

$$\beta_t \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t \delta_{1,t}(\alpha_e + \beta_{t+1}\delta_{2,t+1}\gamma_e\alpha_e)}{I_t} = \frac{K_t}{I_t} \tag{47}$$

For period 1:

$$\begin{aligned}\beta V_{t+1}(\Psi_{t+1}) = \beta_t(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta_{t+1}\beta_t(u(c_{t+2}) + \alpha_e v(\Psi_{t+2})) \\ + \beta_{t+2}\beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(\Psi_{t+3}) + \zeta \ln(a_{t+3}))\end{aligned} \tag{48}$$

Resulting in:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(u(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})) + \beta_{t+1}\beta_t(u(c_{t+2}) \\ & + \alpha_e \ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})) \\ & + \beta_{t+2}\beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(Z_{\theta,t+2})I_{t+2}^{\delta_{1,t+2}}(\theta_t(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})^{\delta_{2,t+2}}) \\ & + \zeta \ln(a_{t+3}))\end{aligned} \tag{49}$$

Giving:

$$\beta \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t \delta_{1,t}(\alpha_e + \beta_{t+1}\delta_{2,t+1}(\alpha_e + \beta_{t+2}\delta_{2,t+2}\gamma_e\alpha_e))}{I_t} = \frac{K_t}{I_t} \tag{50}$$

Using the FOCs for c_t and I_t, and the values above for K_t, results in:

$$\frac{\partial \mathcal{L}}{\partial I_t} = \frac{K_t}{I_t} - u'(c,t)\Lambda_t = 0 \tag{51}$$

Now to derive an optimal solution for I_t, I use:

$$c_t = -\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t \tag{52}$$

plugging in:

$$
\begin{aligned}
\frac{K_t}{I_t} - \frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= 0 \\
\frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= \frac{K_t}{I_t} \\
(-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t \\
(-a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t + K_t \Lambda_t I_t
\end{aligned}
\tag{53}
$$

Thus, the optimal solution for I_t:

$$
I_t = \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{54}
$$

This solution can also be used for period 1, as $I_t = n_t$ and $\Lambda_t = p_{n,t}$. For the borrowing constrained case, $a_{t+1} = a_{min,t}$, for the non-borrowing constrained case, an optimal solution for a_{t+1} is needed, which is derived in section A.5. If $a_t = 0$ and there are no assets, the amount of I_t depends apart from the parameters and related characteristics only on household income y_t.

Optimal solution for s_t and n_t

With I_t one can derive n_t and s_t:

$$
I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}} = [a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{55}
$$

using equation 86 for I_t:

$$
\frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)} = [a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{56}
$$

$$
s_t = \frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{57}
$$

With equation 28:

$$
n_t = \Phi_1 \frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{58}
$$

Optimal solution for a_{t+1} and n_t

From the FOC of the optimization problem, one can use:

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \xi_t + \mathbb{1}\{t < T\}(\lambda_{t+1}\beta_t(1 + r_{t+1})) + \mathbb{1}\{t = T\}\beta_t \frac{\partial V_{T+1}}{\partial a_{T+1}} \tag{59}$$

If the household is not borrowing constraint: $\xi_t = 0$. For period 3:
Equation 59 results in:

$$\frac{1}{-\Lambda_t I_t - a_{t+1} + (1 + r_t)a_t + y_t} = \beta_t \zeta \frac{1}{a_{t+1}} \tag{60}$$

Plugging in the optimal solution for I_t in equation 86:

$$\begin{aligned}
\beta_t\zeta(-\frac{K_t(-a_{t+1} + (1 + r_t)a_t + y_t)}{(1 + K_t)} - a_{t+1} + (1 + r_t)a_t + y_t)) &= a_{t+1} \\
\frac{\beta_t\zeta}{K_t + 1}(-a_{t+1} + (1 + r_t)a_t + y_t) &= a_{t+1} \\
a_{t+1} + \frac{\beta_t\zeta}{K_t + 1}a_{t+1} &= \frac{\beta\zeta}{K_t + 1}((1 + r_t)a_t + y_t)
\end{aligned}$$

Follows:

$$a_{t+1} = \frac{\beta_t\zeta}{(1 + \beta_t\zeta + K_t)}((1 + r_t)a_t + y_t) \tag{61}$$

And for I_t:

$$I_t = \frac{K_t(-(\frac{\beta_t\zeta}{(1+\beta_t\zeta+K_t)}((1 + r_t)a_t + y_t)) + (1 + r_t)a_t + y_t)}{\Lambda_t(1 + K_t)} \tag{62}$$

Which leads to:

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t + \zeta\beta_t)}((1 + r_t)a_t + y_t) \tag{63}$$

For period 2:

$$\lambda_t = \lambda_{t+1}\beta_t(1 + r_{t+1})$$

$$-\Lambda_{t+1}I_{t+1} - a_{t+2} + (1 + r_{t+1})a_{t+1} + y_{t+1} = \beta_t(1 + r_{t+1})(-\Lambda_t I_t - a_{t+1} + (1 + r_t)a_t + y_t)$$

$$\begin{aligned}
&-(\frac{K_{t+1}((1 + r_{t+1})a_{t+1} + y_{t+1})}{(1 + K_{t+1} + \beta_{t+1}\zeta)}) - a_{t+2} + (1 + r_{t+1})a_{t+1} + y_{t+1} = \\
&\qquad \beta_t(1 + r_{t+1})(-(\frac{K_t(-a_{t+1} + (1 + r)a_t + y_t)}{(1 + K_t)}) - a_{t+1} + (1 + r_t)a_t + y_t)
\end{aligned} \tag{64}$$

Plugging in a_{t+2} and $A = (1 + \beta_{t+1}\zeta + K_{t+1})$:

$$-(\frac{K_{t+1}}{A}((1 + r_{t+1})a_{t+1} + y_{t+1}) + \frac{1 + K_{t+1}}{A}(1 + r_{t+1})a_{t+1} + y_{t+1}) = \beta_t(1 + r_{t+1})\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t) \quad (65)$$

$$\frac{1}{A}((1 + r_{t+1})a_{t+1} + y_{t+1}) = \beta_t(1 + r_{t+1})\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t) \quad (66)$$

$$\frac{1}{A}(a_{t+1} + \frac{y_{t+1}}{(1 + r_{t+1})}) = \beta_t\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t)$$
$$\frac{1}{A}(a_{t+1}) + \frac{\beta_t}{1 + K_t}a_{t+1} = -\frac{1}{A}\frac{y_{t+1}}{(1 + r_{t+1})} + \frac{\beta_t}{1 + K_t}((1 + r_t)a_t + y_t))$$

Follows:

$$a_{t+1} = \frac{\beta_t A}{1 + K_t + \beta_t A}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{1 + K_t + \beta_t A}\frac{y_{t+1}}{(1 + r_{t+1})} \quad (67)$$

Plugging in optimal solutions leads to:

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t + \beta_t A)}((1 + r_t)a_t + y_t + \frac{y_{t+1}}{(1 + r_{t+1})}) \quad (68)$$

For period 1, following a similar strategy as in period 2, this yields, with $B = (1 + K_{t+1} + \beta_{t+1}(1 + \beta_{t+2}\zeta + K_{t+2}))$:

$$a_{t+1} = \frac{\beta_t B}{1 + K_t + \beta_t B}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{1 + K_t + \beta_t B}(\frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+1}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (69)$$

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t + \beta_t B)}((1 + r_t)a_t + y_t + \frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+1}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (70)$$

Regarding borrowing constraints, individuals can be never constraint, which is the solution above. Otherwise, they can be constrained always or any combination of order of constrained and unconstrained periods. Exemplary, see here the solution for borrowing constraint in period 3 only:

For period 3:

$$a_{t+1} = a_{min} \tag{71}$$

and

$$I_t = \frac{K_t}{\Lambda_t(1+K_t)}((1+r_t)a_t + y_t - a_{min}) \tag{72}$$

For period 2, with $C = 1 + Kt + \beta_t(1 + K_{t+1})$:

$$a_{t+1} = \frac{\beta_t(1+K_{t+1})}{C}((1+r_t)a_t + y_t) - \frac{1+K_t}{C}\frac{y_{t+1} - a_{min}}{1+r_{t+1}} \tag{73}$$

$$I_t = \frac{K_t}{\Lambda_t C}((1+r_t)a_t + y_t + \frac{y_{t+1} - a_{min}}{1+r_{t+1}}) \tag{74}$$

For period 1, with $D = 1 + K_t + \beta_t(1 + K_{t+1} + \beta_{t+1}(1 + K_{t+2}))$:

$$a_{t+1} = \frac{\beta_t C}{D}((1+r_t)a_t + y_t) - \frac{1+K_t}{D}(\frac{y_{t+1}}{(1+r_{t+1})} + \frac{y_{t+2} - a_{min}}{(1+r_{t+1})(1+r_{t+2})}) \tag{75}$$

$$I_t = \frac{K_t}{\Lambda_t D}((1+r_t)a_t + y_t + \frac{y_{t+1}}{(1+r_{t+1})} + \frac{y_{t+2} - a_{min}}{(1+r_{t+1})(1+r_{t+2})}) \tag{76}$$

Similar pathways can be constructed for households being borrowing constraint in period 2 and 1.

If constrained in period 3 when calculation period 2:

$$\lambda_t = \lambda_{t+1}\beta_t(1+r_{t+1})$$

$$-\Lambda_{t+1}I_{t+1} - a_{t+2} + (1+r_{t+1})a_{t+1} + y_{t+1} = \beta_t(1+r_{t+1})(-\Lambda_t I_t - a_{t+1} + (1+r_t)a_t + y_t)$$

$$\begin{aligned} &-(\frac{K_{t+1}((1+r_{t+1})a_{t+1} + y_{t+1})}{(1+K_{t+1}+\beta_{t+1}\zeta)}) - a_{t+2} + (1+r_{t+1})a_{t+1} + y_{t+1} = \\ &\qquad \beta_t(1+r_{t+1})(-(\frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{(1+K_t)}) - a_{t+1} + (1+r_t)a_t + y_t) \end{aligned} \tag{77}$$

Plugging in $a_{t+2} = a_{min}$ and $A = (1 + \beta_{t+1}\zeta + K_{t+1})$:

$$\begin{aligned} &-(\frac{K_{t+1}((1+r_{t+1})a_{t+1} + y_{t+1})}{A} - a_{min} + (1+r_{t+1})a_{t+1} + y_{t+1} = \\ &\qquad \beta_t(1+r_{t+1})(-(\frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{(1+K_t)}) - a_{t+1} + (1+r_t)a_t + y_t) \end{aligned} \tag{78}$$

$$\frac{(1+\beta_{t+1}\zeta)((1+r_{t+1})a_{t+1}+y_{t+1})}{A}-a_{min}= \\ \beta_t(1+r_{t+1})\frac{(-a_{t+1}+(1+r)a_t+y_t)}{(1+K_t)} \quad (79)$$

$$\frac{(1+\beta_{t+1}\zeta)((1+r_{t+1})a_{t+1})}{A}+\beta_t(1+r_{t+1})\frac{a_{t+1}}{(1+K_t)}= \\ \beta_t(1+r_{t+1})\frac{((1+r)a_t+y_t)}{(1+K_t)}-\frac{(1+\beta_{t+1}\zeta)(y_{t+1})}{A}+a_{min} \quad (80)$$

$$\frac{(1+\beta_{t+1}\zeta)(a_{t+1})}{A}+\frac{\beta_t a_{t+1}}{(1+K_t)}= \\ \beta_t\frac{((1+r)a_t+y_t)}{(1+K_t)}-\frac{(1+\beta_{t+1}\zeta)(y_{t+1})}{A(1+r_{t+1})}+\frac{a_{min}}{(1+r_{t+1})} \quad (81)$$

$$\frac{(1+K_t)(1+\beta_{t+1}\zeta)(a_{t+1})}{A(1+K_t)}+\frac{A\beta_t a_{t+1}}{A(1+K_t)}= \\ \beta_t\frac{((1+r)a_t+y_t)}{(1+K_t)}-\frac{(1+\beta_{t+1}\zeta)(y_{t+1})}{A(1+r_{t+1})}+\frac{a_{min}}{(1+r_{t+1})} \quad (82)$$

$$(1+K_t)(1+\beta_{t+1}\zeta)(a_{t+1})+A\beta_t a_{t+1}= \\ A\beta_t((1+r)a_t+y_t)-\frac{(1+Kt)(1+\beta_{t+1}\zeta)(y_{t+1})}{(1+r_{t+1})}+\frac{A(1+K_t)a_{min}}{(1+r_{t+1})} \quad (83)$$

$$(((1+K_t)(1+\beta_{t+1}\zeta))+A\beta_t)a_{t+1}= \\ A\beta_t((1+r)a_t+y_t)-\frac{(1+Kt)(1+\beta_{t+1}\zeta)(y_{t+1})}{(1+r_{t+1})}+\frac{A(1+K_t)a_{min}}{(1+r_{t+1})} \quad (84)$$

with $E=(((1+K_t)(1+\beta_{t+1}\zeta))+A\beta_t)$

$$a_{t+1}= \\ \frac{A\beta_t}{E}((1+r)a_t+y_t)-\frac{(1+Kt)(1+\beta_{t+1}\zeta)}{E}\frac{(y_{t+1})}{(1+r_{t+1})}+\frac{A(1+K_t)}{E}\frac{a_{min}}{(1+r_{t+1})} \quad (85)$$

For I_t:

$$I_t = \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{86}$$

$$I_t = \frac{K_t(-(\frac{A\beta_t}{E}((1+r)a_t + y_t) - \frac{(1+Kt)(1+\beta_{t+1}\zeta)}{E}\frac{(y_{t+1})}{(1+r_{t+1})} + \frac{A(1+K_t)}{E}\frac{a_{min}}{(1+r_{t+1})}) + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{87}$$

Optimal solution for c_t

If values for I_t, by that s_t and n_t, and a_{t+1} are determined, the optimal c_t simply is:

$$c_t = (1+r)a_t + y_t - p_{n,t}n_t - p_{s,t}s_t - a_{t+1} \tag{88}$$

Optimal solution if n_t is constrained

To derive the relative demands we take first-order conditions for the minimization problem:

$$\begin{aligned} \min_{n_t, s_t} \quad & p_{n,t}n_t + p_{s,t}s_t \\ \text{s.t. } & n_t \leq 5 \\ & It = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{89}$$

The Lagrangian looks the following:

$$\mathcal{L} = p_{n,t}n_t + p_{s,t}s_t - \lambda_{1,t}(I_t - [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}) - \lambda_{2,t}(n_t - 5) \tag{90}$$

Deriving first order conditions in period 2 and 3:

$$\frac{\partial \mathcal{L}}{\partial s_t} = p_{s,t} - \lambda_{1,t}(a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t - 1}\rho_t) \times [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t} - 1}\frac{1}{\rho_t} = 0 \tag{91}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = p_{n,t} - \lambda_{1,t}(n_t^{\rho_t - 1}\rho_t) \times [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t} - 1}\frac{1}{\rho_t} - \lambda_{2,t} = 0 \tag{92}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{1,t}} = I_t - [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} = 0 \tag{93}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{2,t}} = n_t - 5 = 0 \tag{94}$$

If constraints are not binding, $\lambda_{2,t} = 0$, since $n_t < 5$. Then see solution above. If they are binding, this means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}$. If I_t is given, it follows:

$$s_t = \left(\frac{(I_t^{\rho_t} - 5^{\rho_t})}{a_{s,t}(Z_{s,t}, \eta)}\right)^{\frac{1}{\rho_t}} \tag{95}$$

In case the household is constrained ($n_t = 5$), this price does not apply, as it uses the fact that, s_t can be expressed as a share of n_t given the level of investments. In the case that $n_t = 5$, therefore, the household maximizes differently (see next section). In period 1 $\Lambda_t = p_{n,t}$ as investment input decisions only take place for nutrition. This means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5]^{\frac{1}{\rho_t}}$

$$\begin{aligned}
V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, s_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\
& + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\
\text{s.t.} \quad c_t + 5p_{n,t} + p_{s,t}s_t + a_{t+1} &= (1 + r)a_t + y_t \\
a_{t+1} &\geq a_{min,t} \\
\text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}} \\
V_{T+1}(\Psi_{T+1}) &= \alpha_e\gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\
u(c_t) &= \ln(c_t) \\
v(\Psi_t) &= \ln(\Psi_t) \\
I_t &= [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}
\end{aligned} \tag{96}$$

Then:

$$\frac{\partial \mathcal{L}}{\partial s_t} = \beta \frac{\partial V_{t+1}}{\partial I_t}\frac{\partial I_t}{\partial s_t} - \lambda_t(p_{s,t}) = 0 \tag{97}$$

Drawing from the non-binding case, therefore:

$$\beta \frac{\partial V_{T+1}}{\partial I_t} = \frac{K_t}{I_t} = \frac{K_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{98}$$

which results in:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}} & (a_{s,t}(Z_{s,t}, \eta)s_t^{(\rho_t - 1)}[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t} - 1}) \\
& - u'(c, t)p_{s_t} = 0
\end{aligned} \tag{99}$$

which yields:

$$u'(c,t)p_{s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]} a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)} \tag{100}$$

Plugging in the budget constraint:

$$\frac{p_{s_t}}{-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]} a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)} \tag{101}$$

yields:

$$0 = p_{s_t}[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}] - K_t a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)}(-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t) \tag{102}$$

which can only be solved numerically.

GMM equations for investment parameters

To derive the relative demand ratios, one goes back to equation 27 and takes logs to get linear equations, using that $a_{s,t}(Z_{s,t},\eta) = exp(\phi_{s,t}Z_{s,t} + \eta)$:

$$\begin{aligned}
\ln(\frac{p_{n,t}}{p_{s,t}}) &= -\phi_{s,t}Z_{s,t} + (\rho_t - 1)\ln(\frac{n_t}{s_t}) - \eta \\
\ln(\frac{n_t}{s_t}) &= \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} - \frac{1}{1-\rho_t}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta
\end{aligned}$$

Adding $\ln(\frac{p_{n,t}}{p_{s,t}})$ to both sides yields:

$$\ln(\frac{p_{n,t}n_t}{p_{s,t}s_t}) = \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta$$

GMM equations for human capital parameters

$$\Psi_{t+1} = \theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}} \tag{103}$$

Using the human capital formation with $\theta_t(Z_{\theta,t}) = \exp(\phi_{\theta,t}Z_{\theta,t})$, taking logs:

$$\ln(\Psi_{t+1}) = \phi_{\theta,t}Z_{\theta,t} + \delta_{1,t}\ln(I_t) + \delta_{2,t}\ln(\Psi_t) \tag{104}$$

Since Ψ_t are latent skills, I assume the underlying measurement system with $S_{hs,t}$ and $S_{ts,t}$, which are observed height and test scores:

$$S_{ts_1,t} = \lambda_{ts_1,t} \ln(\Psi_t) + \epsilon_{ts_1,t} \tag{105}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t} \ln(\Psi_t) + \epsilon_{ts_2,t} \tag{106}$$

Since height is observed in all periods, I can normalize $\lambda_{ts_1} = 1$ to allow for comparability of measures (see Cunha, Heckman and Schennach (2010)).

Replacing the latent skills with the measurements leads too:

$$S_{ts_1,t+1} = \phi_{\theta,t} Z_t + \delta_{1,t} \ln(I_t) + \delta_{2,t} S_{ts_1} \tag{107}$$

and:

$$\frac{1}{\lambda_{ts_2,t+1}} S_{ts_2,t+1} = \phi_{\theta,t} Z_t + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts_2,t}} S_{ts_2} \tag{108}$$

To identify $\lambda_{ts_2,t}$ further equations are needed. To get these I exploit the covariance structure, similar to (Cunha, Heckman and Schennach, 2010). One can replace Ψ_t in equation 105 with using equation 105:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_2,t}, S_{ts_1,t+1})} = \lambda_{ts_2,t} \tag{109}$$

and:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_1,t}, S_{ts_2,t+1})} = \lambda_{ts_2,t+1} \tag{110}$$

Using that these measures have mean 0, the covariance can be rearanged to:

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1} S_{ts_2,t+1}) S_{ts_1,t}] \tag{111}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t} S - \lambda_{ts_2,t} S_{ts_2,t}) S_{ts_1,t+1}] \tag{112}$$

A.6 Additional tables

Table XII: Distribution of parenting skill types η by total amount of types

	Observations for type:				
Amount of types	Type 0	Type 1	Type 2	Type 3	Type 4
K=2	4,417	2,020			
K=3	2,990	2,833	614		
K=4	2,956	2,813	391	277	
K=5	2,664	547	2,863	9	354

Note: This table summarizes the amount of observation for each set of types, for different total amount of types secified.

Table XIII: Policy counterfactuals - investment change

	Cash transfer	Nutrition subsidy	Schooling subsidy	Cash+ nutrition	Cash+ schooling	Nutrition+ schooling
Change in mean investments (%):						
Primary school	1.52	16.57	4.49	17.77	5.34	20.35
High school	1.77	16.02	13.04	17.34	15.42	32.31

Note: Policies are designed to have the same costs (in 100,000 rupees ∼ \$7), resulting in a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

Table XIV: Policy counterfactuals by income decile

Income decile:	1	2	3	4	5	6	7	8	9	10
Change in mean skills (SD):										
Cash	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Nutrition	0.04	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02
Schooling	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02
Change in mean investements (%):										
Cash	1.97	1.40	0.69	0.62	0.75	0.44	0.46	0.30	0.24	0.03
Nutrition	17.65	15.23	13.47	13.58	11.84	10.68	10.52	8.59	8.94	5.40
Schooling	8.02	9.54	7.66	8.73	8.39	8.79	9.13	8.96	9.25	6.77
Cost by 0.01 SD increase per child:										
Cash	1.42	2.04	2.47	3.15	2.81	5.54	5.91	5.84	14.21	61.71
Nutrition	0.18	0.24	0.30	0.33	0.45	0.37	0.51	0.64	0.71	1.29
Schooling	0.24	0.33	0.45	0.55	0.60	0.67	0.90	1.02	1.40	3.94

Note: Costs are expressed in 100,000,000 rupees (∼ \$0,007), simulated are a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

Table XV: Estimation results for skill formation parameters for 2 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.10	(0.65)***	-10.12	(4.16)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-2.24	(0.39)***	-35.08	(12.33)***
Mother primary			0.88	(0.19)***	2.58	(1.02)**
Mother high			1.51	(0.30)***	4.14	(1.62)**
Father primary			0.01	(0.14)	0.38	(0.38)
Father high			-0.18	(0.17)	0.20	(0.41)
Age			-0.04	(0.04)	2.80	(1.05)***
Female			0.06	(0.11)	1.21	(0.52)**
Rural area			-2.27	(0.41)***	-4.47	(1.74)**
No. of siblings			-0.61	(0.11)***	-1.90	(0.71)***
Mother not Islam			0.32	(0.19)*	1.35	(0.67)**
Parenting type 1			-1.53	(0.29)***	-3.13	(1.23)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.26	(0.10)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.12	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

TABLE XVI: Estimation results for skill formation parameters for 3 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.37	(0.74)***	-10.37	(4.36)**
Implied elasticity			0.23		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.47)***	-38.98	(14.12)***
Mother primary			1.01	(0.22)***	2.84	(1.14)**
Mother high			1.71	(0.34)***	4.54	(1.81)**
Father primary			0.05	(0.15)	0.51	(0.41)
Father high			-0.12	(0.17)	0.37	(0.44)
Age			-0.05	(0.04)	2.89	(1.11)***
Female			0.03	(0.12)	1.19	(0.53)**
Rural area			-2.44	(0.46)***	-4.75	(1.89)**
No. of siblings			-0.67	(0.12)***	-1.98	(0.76)***
Mother not Islam			0.33	(0.20)	1.43	(0.71)**
Parenting type 1			0.14	(0.13)	0.03	(0.31)
Parenting type 2			3.49	(0.68)***	6.45	(2.55)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.22	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.25	(0.09)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

Table XVII: Estimation results for skill formation parameters for 5 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.19	(0.68)***	-9.81	(3.92)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.61	(0.44)***	-37.26	(12.76)***
Mother primary			0.98	(0.21)***	2.71	(1.04)***
Mother high			1.61	(0.31)***	4.39	(1.66)***
Father primary			0.06	(0.14)	0.54	(0.40)
Father high			-0.12	(0.17)	0.39	(0.42)
Age			-0.04	(0.04)	2.75	(1.00)***
Female			0.04	(0.11)	1.10	(0.48)**
Rural area			-2.37	(0.43)***	-4.63	(1.74)***
No. of siblings			-0.64	(0.11)***	-1.89	(0.69)***
Mother not Islam			0.36	(0.19)*	1.41	(0.67)**
Parenting type 1			1.52	(0.35)***	2.32	(1.01)**
Parenting type 2			-0.04	(0.12)	0.36	(0.33)
Parenting type 3			-0.04	(2.44)	16.02	(7.23)**
Parenting type 4			4.25	(0.82)***	8.36	(3.17)***
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.07	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.27	(0.09)***
Mother primary	0.06	(0.04)	0.07	(0.04)*	0.06	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.13	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.98	(0.11)	1.07	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.27	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

Table XVIII: Robustness check: GMM without constrained individuals

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.33	(0.76)***	-14.60	(8.71)*
Implied elasticity			0.23		0.06	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.42	(0.48)***	-53.02	(28.45)*
Mother primary			1.10	(0.24)***	4.10	(2.34)*
Mother high			1.78	(0.37)***	7.24	(4.12)*
Father primary			0.23	(0.16)	0.79	(0.69)
Father high			0.05	(0.19)	0.25	(0.62)
Age			-0.04	(0.04)	4.05	(2.26)*
Female			0.02	(0.13)	1.53	(0.94)
Rural area			-2.36	(0.46)***	-6.64	(3.80)*
No. of siblings			-0.68	(0.13)***	-2.71	(1.51)*
Mother not Islam			0.21	(0.21)	2.06	(1.34)
Parenting type 1			-0.37	(0.15)**	-0.68	(0.56)
Parenting type 2			4.26	(0.90)***	12.26	(6.99)*
Parenting type 3			1.62	(0.52)***	2.93	(1.99)
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.17	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.21	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.